Student: _____
Date: _____

Instructor: Vignon Oussa
Course: [Spring 24] Multivariable Calculus (Dr. **Assignment:** Double Integration Part 2 Oussa)

1. Change the order of integration in the integral $\int_0^1 \int_{y^2}^{\sqrt{y}} f(x,y)\,dx\,dy$.

Reverse the order of integration.

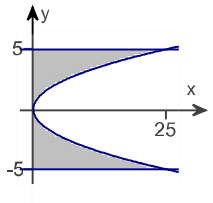
$\int \int f(x,y)\,dy\,dx$

(Type exact answers.)

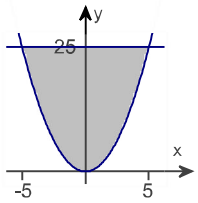
2. Sketch the region of integration for $\int_{-5}^{5} \int_{x^2}^{25} xy\,dy\,dx$.

Choose the correct sketch of the region below.

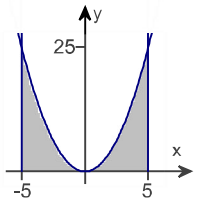
A.



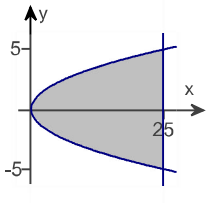
B.



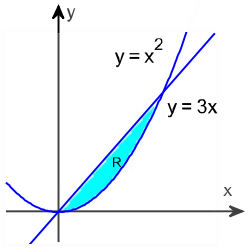
C.



D.



3. Consider the region R shown in the figure and write an iterated integral of a continuous function f over R.



Choose the correct iterated integral below.

A.

$\int_{3x}^{x^2} \int_{0}^{3} f(x,y)\,dy\,dx$

B.

$\int_{0}^{3} \int_{x^2}^{3x} f(x,y)\,dx\,dy$

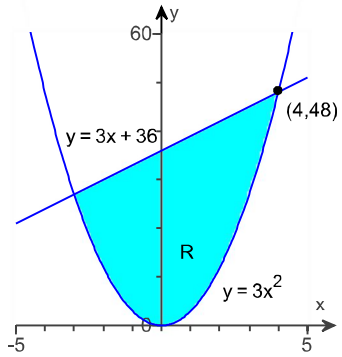
C.

$\int_{0}^{3} \int_{x^2}^{3x} f(x,y)\,dy\,dx$

D.

$\int_{x^2}^{3x} \int_{0}^{3} f(x,y)\,dx\,dy$

4. Consider the region shown in the figure to the right and write an iterated integral of a continuous function over R.



Choose the correct answer below.

- **A.** $\int_{3x^2}^{3x+36} \int_{-3}^{4} f(x,y)\, dydx$
- **B.** $\int_{0}^{48} \int_{3x^2}^{3x+36} f(x,y)\, dxdy$
- **C.** $\int_{3x^2}^{3x+36} \int_{-3}^{4} f(x,y)\, dxdy$
- **D.** $\int_{-3}^{4} \int_{3x^2}^{3x+36} f(x,y)\, dydx$

5. Evaluate the following integral as it is written.

$$\int_{\pi/4}^{2\pi/3} \int_{\cos x}^{\sin x} dy\, dx$$

$\int_{\pi/4}^{2\pi/3} \int_{\cos x}^{\sin x} dy\, dx =$ [ ]

(Type an exact answer, using radicals as needed.)

6. Evaluate the double integral.

$$\int_{0}^{1} \int_{-3x}^{x^2} y\, dy\, dx$$

$\int_{0}^{1} \int_{-3x}^{x^2} y\, dy\, dx =$ [ ] (Type an exact answer.)

7. Evaluate the following integral.

$$\int_{0}^{\frac{\pi}{2}} \int_{0}^{2y\cos y} dx\, dy$$

$\int_{0}^{\frac{\pi}{2}} \int_{0}^{2y\cos y} dx\, dy =$ [ ] (Type an exact answer, using π as needed.)

8. Evaluate the following integral as written.

$$\int_0^4 \int_{-\sqrt{16-y^2}}^{\sqrt{16-y^2}} 3xy \, dx \, dy$$

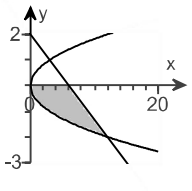
Evaluate the integral.

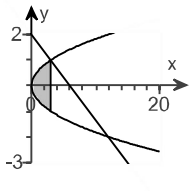
$$\int_0^4 \int_{-\sqrt{16-y^2}}^{\sqrt{16-y^2}} 3xy \, dx \, dy = \square$$ (Simplify your answer.)

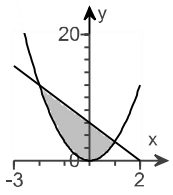
9. Sketch the following region and write an iterated integral of a continuous function f over the region.

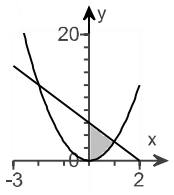
$R = \{(x,y): 0 \le x \le 1, 3x^2 \le y \le -3x+6\}$

Choose the correct sketch of the region below.

A. 

B. 

C. 

D. 

Choose the correct iterated integral below.

A. $\int_0^1 \int_{3x^2}^{-3x+6} f(x,y) \, dy \, dx$

B. $\int_0^{3x^2} \int_0^1 f(x,y) \, dy \, dx$

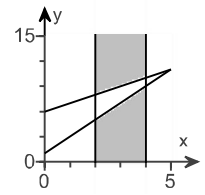
C. $\int_{-2}^1 \int_{-3x+6}^{3x^2} f(x,y) \, dx \, dy$

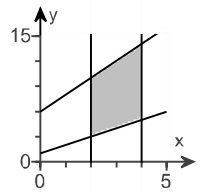
D. $\int_{3x^2}^{-3x+6} \int_{-2}^1 f(x,y) \, dx \, dy$

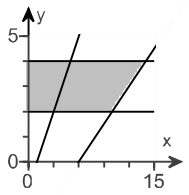
10. Sketch the following region and write an iterated integral of a continuous function f over the region.

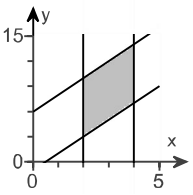
$R = \{(x,y): 2 \le x \le 4, x+1 \le y \le 2x+6\}$

Choose the correct sketch of the region below.

A. 

B. 

C. 

D. 

Choose the correct iterated integral below.

A. $\int_{x+1}^0 \int_0^4 f(x,y) \, dy \, dx$

B. $\int_2^4 \int_{x+1}^{2x+6} f(x,y) \, dy \, dx$

C. $\int_{x+1}^{2x+6} \int_2^4 f(x,y) \, dx \, dy$

D. $\int_0^4 \int_{2x+6}^{x+1} f(x,y) \, dx \, dy$

11. Evaluate the following integral. A sketch is helpful.

$\iint_R 8xy\ dA$; where R is bounded by $y = 4 - 2x$, $y = 0$, and $x = 4 - \frac{y^2}{4}$ in the first quadrant

$\iint_R 8xy\ dA =$ [ ] (Type an exact answer.)

12. Evaluate the following integral. A sketch is helpful.

$\iint_R 3x^2\ dA$; where R is bounded by $y = 0$, $y = 8x + 16$, and $y = 4x^3$

$\iint_R 3x^2\ dA =$ [ ] (Type an exact answer.)

13. Evaluate the following integral.

$\iint_R 6x\ \sec^2 y\ dA$; $R = \left\{(x,y): 0 \le y \le x^2, 0 \le x \le \frac{\sqrt{\pi}}{2}\right\}$

$\iint_R 6x\ \sec^2 y\ dA =$ [ ] (Type an exact answer.)

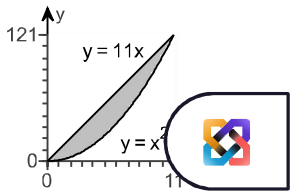
14. Evaluate the following integral.

$\iint_R \frac{xy}{1+x^2+y^2} dA$; $R = \{(x,y): 0 \le y \le x, 0 \le x \le 1\}$

$\iint_R \frac{xy}{1+x^2+y^2} dA =$ [ ] (Type an exact answer.)

15. Reverse the order of integration in the following integral.

$\int_0^{121} \int_{y/11}^{\sqrt{y}} f(x,y)\ dx\ dy$



Choose the correct reversed integral below.

A. $\int_{x^2}^{11x} \int_0^{11} f(x,y)\ dy\ dx$

B. $\int_0^{11} \int_{x^2}^{11x} f(x,y)\ dy\ dx$

C. $\int_0^{121} \int_{x^2}^{11x} f(x,y)\ dy\ dx$

D. $\int_{x^2}^{11x} \int_0^{11} f(x,y)\ dx\ dy$

16. Find the volume of the following solid.

The tetrahedron bounded by the coordinate planes ($x = 0$, $y = 0$, $z = 0$) and the plane $5x + 6y + z - 30 = 0$

The volume is ☐ (1) _____
(Type an exact answer.)

(1) ○ cubic units.
○ units.
○ square units.

---

17. Find the volume of the following solid.

The solid in the first octant bounded by the coordinate planes and the surface $z = 12 - y - 3x^2$

The volume of the solid is ☐ (1) _____
(Type an exact answer.)

(1) ○ cubic units.
○ units.
○ square units.

---

18. Find the volume of the following solid.

The segment of the cylinder $x^2 + y^2 = 1$ bounded above by the plane $z = 13 + x + y$ and below by $z = 0$

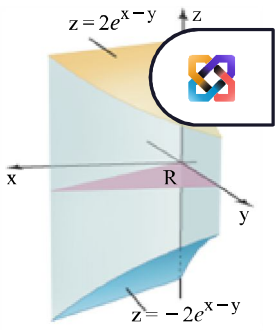
The volume of the region is ☐ (1) _____
(Type an exact answer.)

(1) ○ units.
○ square units.
○ cubic units.

---

19. Find the volume of the following solid.

The solid S between the surfaces $z = 2e^{x-y}$ and $z = -2e^{x-y}$, where S intersects the xy-plane in the region $R = \{(x,y): 0 \le x \le y, 0 \le y \le 3\}$



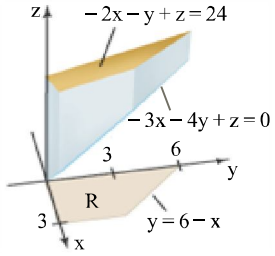
The volume of the solid is ☐ (1) _____
(Type an exact answer.)

(1) ○ units.
○ cubic units.
○ square units.

---

20. Find the volume of the following solid.

The solid above the region $R = \{(x,y): 0 \le x \le 3,\ 0 \le y \le 6 - x\}$ and between the planes $-3x - 4y + z = 0$ and $-2x - y + z = 24$

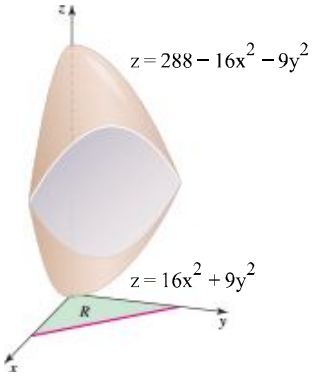


The volume of the solid is ☐ (1) _____
(Type an exact answer.)

(1) ○ square units.
○ cubic units.
○ units.

21. Find the volume of the following solid.

The solid above the region $R = \{(x,y): 0 \le x \le 3,\ 0 \le y \le 4 - \frac{4}{3}x\}$ bounded by the paraboloids $z = 16x^2 + 9y^2$ and $z = 288 - 16x^2 - 9y^2$ and the coordinate planes in the first octant



The volume of the region is ☐ (1) _____
(Type an exact answer.)

(1) ○ square units.
○ units.
○ cubic units.

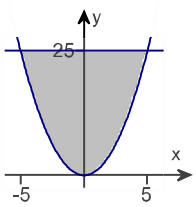
1. 0

   1

   x^2

   $\sqrt{x}$

---

2. 

   B.

---

3. $\int_0^3 \int_{x^2}^{3x} f(x,y)\, dy\, dx$

   C.

---

4. $\int_{-3}^{4} \int_{3x^2}^{3x+36} f(x,y)\, dy dx$

   D.

---

5. $\sqrt{2} - \frac{\sqrt{3}-1}{2}$

---

6. $-\frac{7}{5}$

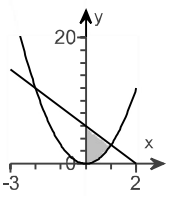
---

7. $\pi - 2$

---

8. 0

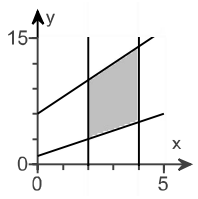
---

9. 

   D.

   A. $\int_0^1 \int_{3x^2}^{-3x+6} f(x,y)\, dy\, dx$

10.



B.

B. $\int_{2}^{4}\int_{x+1}^{2x+6} f(x,y)\,dy\,dx$

---

11. $\frac{448}{3}$

---

12. 128

---

13. $3\ln\left(\sqrt{2}\right)$

---

14. $\frac{1}{8}(3\ln(3)-4\ln(2))$

---

15. B. $\int_{0}^{11}\int_{x^2}^{11x} f(x,y)\,dy\,dx$

---

16. 150

(1) cubic units.

---

17. $\frac{384}{5}$

(1) cubic units.

---

18. 13π

(1) cubic units.

---

19. $8+\frac{4}{e^3}$

(1) cubic units.

---

20. $\frac{423}{2}$

(1) cubic units.

---

21. 1152

(1) cubic units.