Definition 1 : $\lim_{n\to\infty} x_n = \infty$ means $\forall M > 0, \exists N \in \mathbb{I}$ such that $n > N \Rightarrow x_n > M$

Definition 2: $\lim_{n\to\infty} x_n = -\infty$ means $\forall M > 0, \exists N \in \mathbb{I}$ such that $n > N \Rightarrow x_n < -M$

Example 1 Prove that $\left(\sqrt{n^2+1}\right)_{n\in I}$ is properly divergent.

Example 2 Prove that $\left(\frac{n}{\sqrt{n+4}}\right)_{n\in I}$ is properly divergent.

Example 3 Prove that $\left(\sin\left(\frac{\pi n}{7}\right)\cdot\left(\frac{n^3-n+1}{2n^2+n+1}\right)\right)_{n\in I}$ is not a properly divergent sequence.

Example 4 Let $(a_n)_{n=1}^{\infty}$ be a properly divergent sequence and let $(y_n)_{n=1}^{\infty}$ be a sequence such that $(x_n \cdot y_n)_{n=1}^{\infty}$ is convergent. Prove that $\lim_{n\to\infty} y_n = 0$.