

IN-CLASS ACTIVITIES FOR APPENDIX A: REVIEW OF BASIC PROOF TECHNIQUES

Problem 1. Answer the following questions

- (1) Write down an example of a false implication and write down a negation of the given implication.
- (2) Write down an example of a implication and its converse. Is it true that an implication and its converse must have the same truth value? Justify your answer.
- (3) Negate the following statement: There exists $x, y, a \in \mathbb{R}$ such that if $x \in A$ then $x < a$ and $x \leq y$.

Problem 2. Give a direct proof of each mathematical statement

- (1) The square of a rational number is rational.
- (2) Suppose that a, b and c are integers. If a^2 divides b and b^3 divides c then a^6 divides c .

Problem 3. Give a proof by **contradiction** of each mathematical statement.

- (1) If n is natural number then $\frac{n}{n+1} > \frac{n}{n+2}$.
- (2) Prove by contradiction that if $0 < x < \frac{\pi}{2}$ then $\sin x + \cos x > 1$.

Problem 4. Use the method of **contrapositive** proof to prove the following statements.

- (1) If $x^2 + 5x < 0$ then x must be negative.
- (2) If $x^2(y + 3)$ is even, then x is even or y is odd.