Section 1.2 Mathematical Induction

- Definition of $\mathbb{N}$ (the set of all natural numbers)
- Well-ordering property of $\mathbb{N}$
- **Principle of Mathematical Induction**. Let S be a subset of $\mathbb{N}$ such that (1) $1 \in S$ and (2) $k \in S \Rightarrow k+1 \in S$. Then $S = \mathbb{N}$
- **Principle of Mathematical Induction ver 2.** Let $n_1 \in \mathbb{N}$ and let $P(n)$ be a statement for each natural number $n \geq n_1$. Suppose that $P(n_1)$ is true and $P(k)$ is true implies that $P(k)$ holds. Then for all $n \geq n_1$, $P(n)$ is true.
- **Examples**
  - Prove that for every natural number n,

$$\sum_{k=1}^{n} k^2 = \frac{n(2n+1)(n+1)}{6}$$

  - Prove that for every natural number n,

$$\sum_{k=1}^{n} k^3 = \frac{1}{4} n^2 (n+1)^2$$

  - Prove that for every natural number n

$$\frac{5^{2n}-1}{8} \in \mathbb{N}$$

In other words, $5^{2n}-1$ is divisible by 8 for all $n \in \mathbb{N}$.

Homework (Section 1.2)

Prove the following assertions

1. For all natural numbers n

$$1^2 + 3^2 + \cdots + (2n-1)^2 = \frac{4n^3 - n}{3}.$$

2. For every natural number n, $n^3 + 5n$ is divisible by 6.

3. For every natural number n, $5^{2n} - 1$ is divisible by 8.

4. For every natural number n, $n < 2^n$.

5. $2^n < n!$ for all natural numbers $n \geq 4$. (Recall that $n! = n\,(n-1)\cdots(2)\,(1)$).

6. For every natural number $n > 1$

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n}.$$