Homework Sec 2.2 (MATH 401)

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1. If $a, b \in \mathbb{R}$, prove that $|a+b| = |a| + |b|$ if and only if $ab \geq 0$.

2. If $a < x < b$ and $a < y < b$, prove that $|x - y| = b - a$. Provide a geometric interpretation to this statement and illustrate its meaning using pictures that make sense to you.

3. Find all real numbers $x \in \mathbb{R}$ such that $|x-1| > |x+1|$. Hint: use the definition of the absolute value function.

4. Show that if a, b are real numbers and a is not equal to b then there exist ϵ-neighborhoods U of a and V of b such that $U \cap V$ is empty.

5. Let x, y be two real numbers. Prove the following.

   (a) $\max\{x, y\} = \dfrac{x + y + |x - y|}{2}$

   (b) $\min\{x, y\} = \dfrac{x + y - |x - y|}{2}$.