

1. Let  $A = \{x \in \mathbb{R} : x > 0\}$ .
  - (a) Does  $A$  have lower bounds?
  - (b) Does  $A$  have upper bounds?
  - (c) Does  $\inf(A)$  exist?
  - (d) Does  $\sup(A)$  exist?
2. Let  $B = \left\{1 - \frac{(-1)^n}{n} : x > 0\right\}$ . Find  $\inf B$  and  $\sup B$
3. Let  $S$  be a nonempty subset of the reals that is bounded below. Prove that the infimum of  $S$  is equal to  $-\sup\{-s : s \in S\}$
4. Let  $S$  be a nonempty subset of the reals. Show that  $u \in \mathbb{R}$  is an upperbound of  $S$  if and only if the conditions  $t \in S$  and  $t > u$  imply that  $t$  is not an element of  $S$ .
5. Let  $X$  be a bounded set of the reals and let  $X_0$  be a nonempty subset of  $X$ . Show that

$$\inf X \leq \inf X_0 \leq \sup X_0 \leq \sup X.$$

6. Let  $X$  be a subset of  $\mathbb{R}$  and suppose that  $x = \sup X$  belongs to  $X$ . If  $u$  is not an element of  $X$ , show that

$$\sup(X \cup \{x\}) = \sup\{x, u\}$$