

Work these problems out

Convert the angle to a decimal in degrees. Round the answer to two decimal places.

1) $55^{\circ}54'33''$

2) $356^{\circ}20'57''$

3) $21^{\circ}17'34''$

Convert the angle to $D^{\circ} M' S''$ form. Round the answer to the nearest second.

4) 172.87°

5) 283.88°

If s denotes the length of the arc of a circle of radius r subtended by a central angle θ , find the missing quantity.

6) $r = 19.99$ centimeters, $\theta = 6.2$ radians, $s = ?$

7) $r = 13.8$ inches, $\theta = 45^{\circ}$, $s = ?$

Solve the problem.

8) For a circle of radius 4 feet, find the arc length s subtended by a central angle of 30° . Round to the nearest hundredth.

Convert the angle in degrees to radians. Express the answer as multiple of π .

9) 30°

Convert the angle in radians to degrees.

10) $-\frac{11\pi}{6}$

Convert the angle in degrees to radians. Express the answer as multiple of π .

11) 144°

If A denotes the area of the sector of a circle of radius r formed by the central angle θ , find the missing quantity. If necessary, round the answer to two decimal places.

12) $r = 3$ feet, $A = 76$ square feet, $\theta = ?$

Solve the problem.

13) A circle has a radius of 5 centimeters. Find the area of the sector of the circle formed by an angle of 10° . If necessary, round the answer to two decimal places.

If A denotes the area of the sector of a circle of radius r formed by the central angle θ , find the missing quantity. If necessary, round the answer to two decimal places.

14) $r = 7$ feet, $A = 74$ square feet, $\theta = ?$

Convert the angle in radians to degrees. Express the answer in decimal form, rounded to two decimal places.

15) 6.54

Convert the angle in degrees to radians. Express the answer in decimal form, rounded to two decimal places.

16) 12°

Convert the angle in radians to degrees.

17) $\frac{11\pi}{4}$

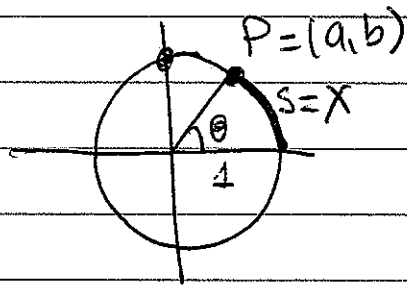
Answer Key

Testname: UNTITLED1

- 1) 55.91°
- 2) 356.35°
- 3) 21.29°
- 4) $172^\circ 52' 12''$
- 5) $283^\circ 52' 48''$
- 6) 123.9 cm
- 7) 10.8 in.
- 8) 2.09 ft
- 9) $\frac{\pi}{6}$
- 10) -330°
- 11) $\frac{4\pi}{5}$
- 12) 16.89 radians
- 13) 2.18 cm^2
- 14) 173.14°
- 15) 374.71°
- 16) 0.21
- 17) 495°

5.2 Trig functions: a unit circle approach

We consider a unit circle (a circle of radius 1)



$$x = r\theta = 1 \cdot \theta = \theta$$

Given a real number x , how do we find the corresponding point P as shown above.

We define a function called the wrapping function such that

$$W(x) = P$$

For example, it is easy to see that

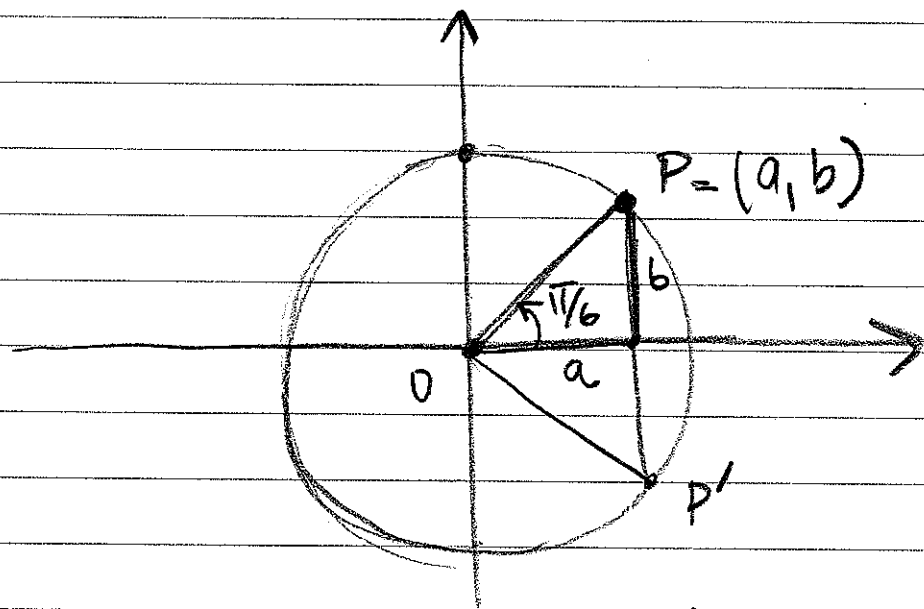
$$W(0) = (1, 0) \quad , \quad W(\pi) = (-1, 0)$$

$$W\left(\frac{\pi}{2}\right) = (0, 1) \quad , \quad W\left(\frac{3\pi}{2}\right) = (0, -1)$$

Question Is the wrapping function one-to-one?

Fact Given any $x \in \mathbb{R}$, in general it is hard to find $W(x)$.

Ex Find $w\left(\frac{\pi}{6}\right)$



Clearly $|OP| = |OP'| = |PP'|$

$$|OP|^2 = a^2 + b^2, \quad b = \frac{1}{2} \text{ since } |PP'| = 1$$

$$1 = a^2 + b^2$$

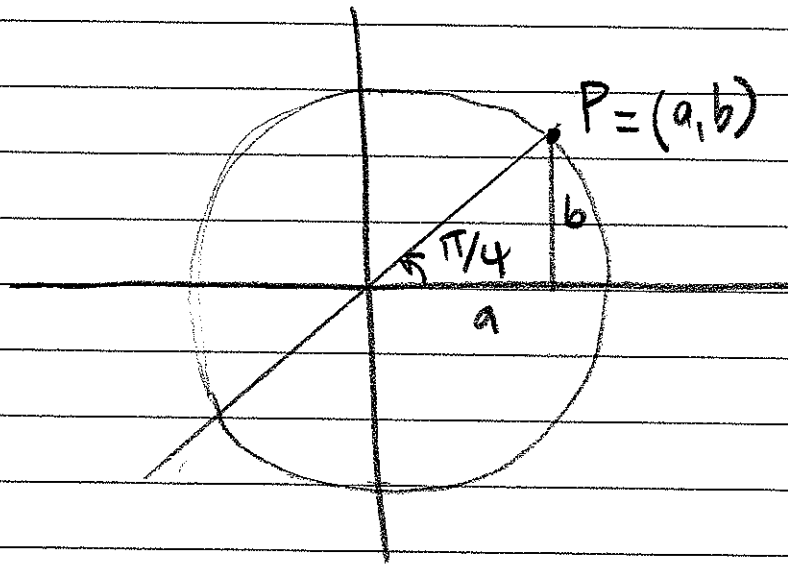
$$1 = a^2 + \left(\frac{1}{2}\right)^2$$

$$a = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

Thus $w\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Ex Find $w(\pi/4)$



$$a^2 + b^2 = 1, \text{ but } a = b$$

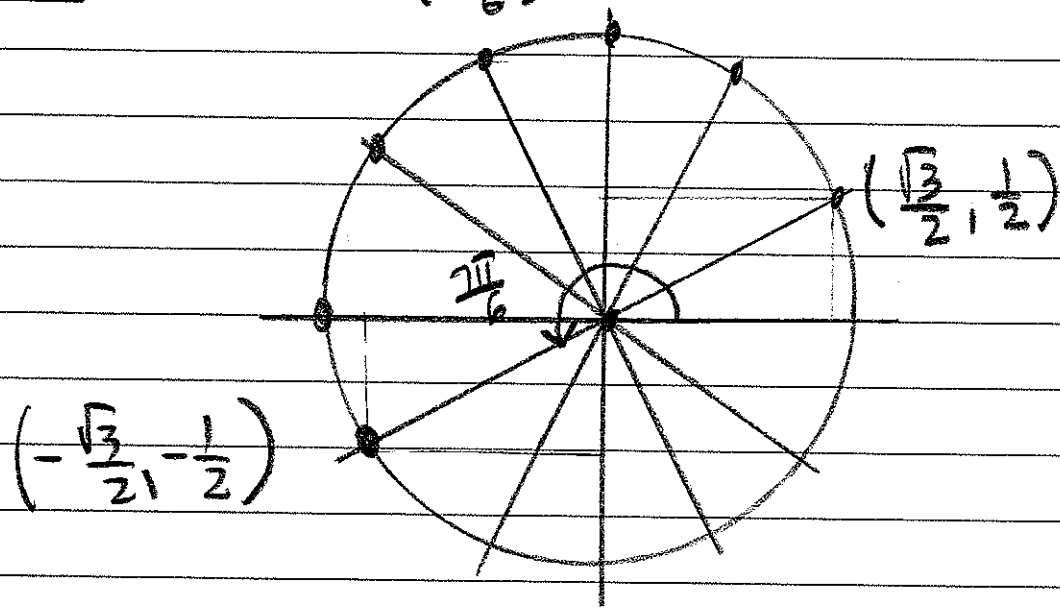
$$a^2 + a^2 = 1$$

$$2a^2 = 1$$

$$a = \frac{\sqrt{2}}{2} = b$$

$$\text{Thus } w\left(\frac{\pi}{4}\right) = P = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

Ex Find $w\left(\frac{7\pi}{6}\right)$



$$w\left(\frac{7\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

Ex Find $w\left(\frac{5\pi}{4}\right)$

Follow above steps.

Key circular points
(Memorize these)

1. $w(0) = (1, 0)$

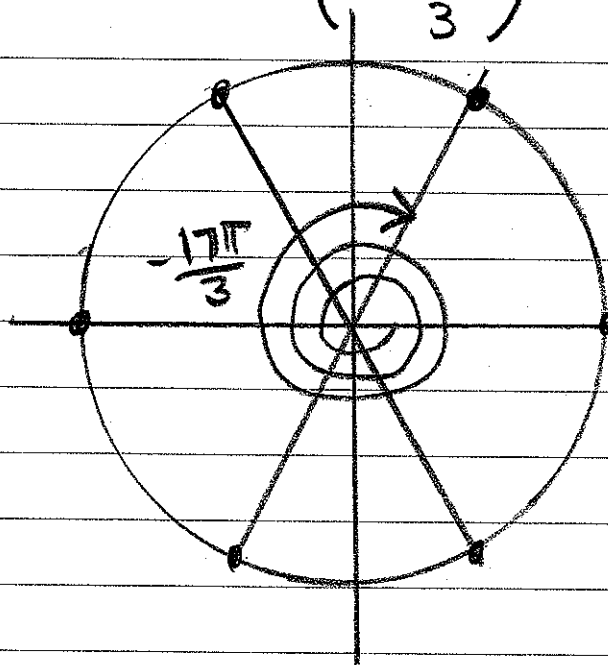
2. $w\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

3. $w\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

4. $w\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

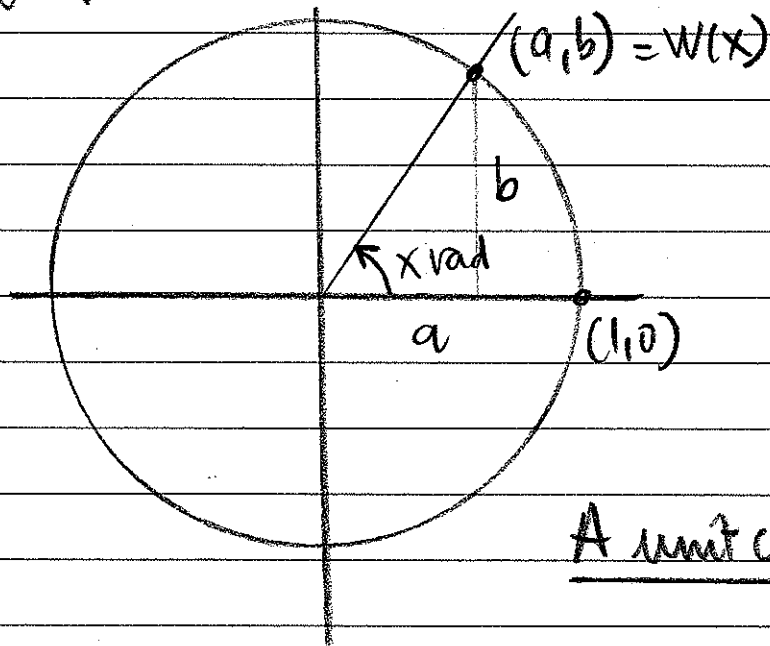
5. $w\left(\frac{\pi}{2}\right) = (0, 1)$

Ex Find $w\left(-\frac{17\pi}{3}\right)$



$$\begin{aligned} w\left(-\frac{17\pi}{3}\right) &= w\left(\frac{\pi}{3}\right) \\ &= \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) \end{aligned}$$

Trig functions



| | |
|----------------------------------|------------------------------------|
| $\sin x = b$ | $\csc x = \frac{1}{b}, b \neq 0$ |
| $\cos x = a$ | $\sec x = \frac{1}{a}, a \neq 0$ |
| $\tan x = \frac{b}{a}, a \neq 0$ | $\cotan x = \frac{a}{b}, b \neq 0$ |

Notice that

$$\sec x = \frac{1}{\cos x}, \cos x \neq 0$$

$$\csc x = \frac{1}{\sin x}, \sin x \neq 0$$

$$\cot x = \frac{1}{\tan x}, \tan x \neq 0$$

Ex Find all 6 trig functions of the angle x if

$$w(x) = \left(\frac{3}{5}, -\frac{4}{5} \right)$$

$$\sin x = -\frac{4}{5}$$

$$\cos x = \frac{3}{5}$$

$$\tan x = \frac{-4/5}{3/5} = -\frac{4}{3}$$

$$\cot x = -\frac{3}{4}$$

$$\sec x = \frac{5}{3}$$

$$\csc x = -\frac{5}{4}$$

Homework P491,

2, 7, 9, 13, 15, 19, 23, 29, 35, 41, 53

125, 127