Exam I Wed 6th

A.7 <u>nth root</u>, <u>Rational Exponent</u>

The principal nth root of a real number a, $n \geq 2$ is denoted $\sqrt[n]{a}$

$$\boxed{\sqrt[n]{a} = b \quad \text{means} \quad a = b^n}$$

$a \geq 0$, $b \geq 0$ if n is even and a, b are real numbers if n is odd.

<u>Ex</u>

$$\sqrt[3]{8} = \sqrt[3]{2^3} = 2$$

$$\sqrt[3]{-64} = \sqrt[3]{(-4)^3} = -4$$

$$\sqrt[4]{(-2)^4} =$$

$n \geq 2$

$\sqrt[n]{a^n} = a$ if n is odd

$\sqrt[n]{a^n} = |a|$ if n is even

$$\sqrt[4]{(-4)^4} = |-4| = 4$$

$$\sqrt[3]{(-6)^3} = -6$$

$$\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

<u>Ex</u> Simplify $\sqrt{32}$

$$\sqrt{32} = \sqrt{16\cdot 2} = \sqrt{16}\,\sqrt{2} = 4\sqrt{2}$$

$$\sqrt[3]{16} = \sqrt[3]{8\times 2} = \sqrt[3]{8}\,\sqrt[3]{2}$$

$$= \sqrt[3]{2^3}\,\sqrt[3]{2}$$

$$= 2\sqrt[3]{2}$$

$$\sqrt[4]{\frac{16x^5}{81}} = \sqrt[4]{\frac{2^4 x^4 x}{3^4}}$$

$$= \frac{\sqrt[4]{2^4 x^4 x}}{\sqrt[4]{3^4}}$$

$$= \frac{\sqrt[4]{2^4}\ \sqrt[4]{x^4}\ \sqrt[4]{x}}{\sqrt[4]{3^4}}$$

$$= \frac{2\,|x|\,\sqrt[4]{x}}{3}$$

Rationalizing the denominator

$$\frac{1\sqrt{3}}{\sqrt{3}\,\sqrt{3}} = \frac{\sqrt{3}}{3}$$

rational

irrational

$$\frac{\sqrt{5}}{\sqrt{3} - 2\sqrt{2}} = \frac{\sqrt{5}\,(\sqrt{3}+2\sqrt{2})}{(\sqrt{3}-2\sqrt{2})(\sqrt{3}+2\sqrt{2})}$$

$$= \frac{\sqrt{5}\sqrt{3} + 2\sqrt{5}\sqrt{2}}{\sqrt{3}\sqrt{3} \cancel{+ 2\sqrt{3}\sqrt{2} - 2\sqrt{3}\sqrt{2}} - 2\sqrt{2}\,2\sqrt{2}}$$

$$= \frac{\sqrt{15} + 2\sqrt{10}}{3 - 4(2)} = \frac{\sqrt{15} + 2\sqrt{10}}{3-8}$$

$$= -\frac{\sqrt{15} + 2\sqrt{10}}{5}$$

$$\frac{5\sqrt[3]{4^2}}{\sqrt[3]{4}\sqrt[3]{4^2}} = \frac{5\sqrt[3]{16}}{\sqrt[3]{4^3}}$$

$$= \frac{5\sqrt[3]{16}}{4}$$

$$= \frac{5\sqrt[3]{8\times 2}}{4}$$

$$= \frac{5\sqrt[3]{8}\sqrt[3]{2}}{4}$$

$$= \frac{5\times 2\sqrt[3]{2}}{4}$$

$$= \boxed{\frac{5\sqrt[3]{2}}{2}}$$

Def

$$\sqrt[n]{a} = a^{\frac{1}{n}} \quad , \quad n \geq 2$$

$$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

Ex

$$4^{3/2} = \sqrt{4^3} = \left(\sqrt{4}\right)^3 = 2^3 = 8$$

$$(-8)^{4/3} = \left(\sqrt[3]{-8}\right)^4 = \left(\sqrt[3]{(-2)^3}\right)^4$$

$$= (-2)^4 = 16$$

Write as a single quotient

$$(x^2+1)^{1/2} + x\frac{1}{\cancel{2}}(x^2+1)^{-1/2}\cdot\cancel{2}x$$

$$= (x^2+1)^{1/2} + x^2(x^2+1)^{-1/2}$$

$$= \frac{(x^2+1)^{1/2}}{1} + \frac{x^2}{(x^2+1)^{1/2}}$$

$$= \frac{(x^2+1)^{1/2}(x^2+1)^{1/2}}{(x^2+1)^{1/2}} + \frac{x^2}{(x^2+1)^{1/2}}$$

$$= \frac{x^2+1+x^2}{(x^2+1)^{1/2}} = \boxed{\frac{2x^2+1}{(x^2+1)^{1/2}}}$$

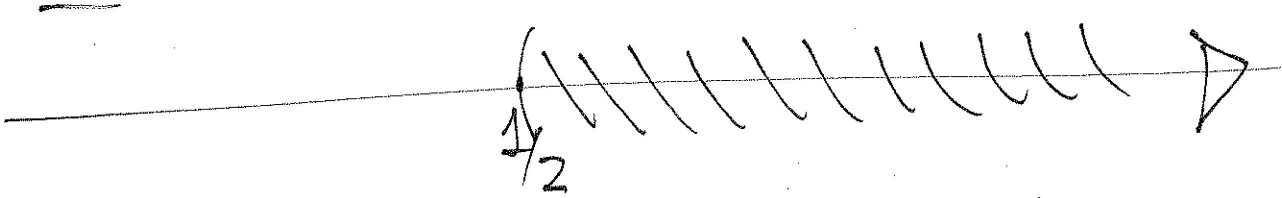
A 10 Interval notation ; Solving Inequalities

A84

Ex $1 \leq x < 4$

$[1, 4)$ Interval notation

Ex $x > \frac{1}{2}$



$(\frac{1}{2}, \infty)$ interval notation.

Ex Solve the inequality

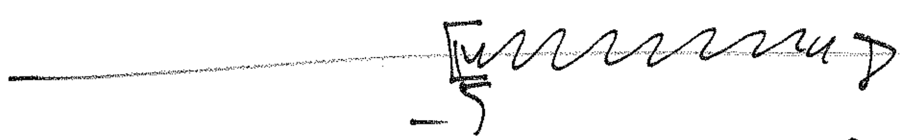
$$4x + 7 \geq 2x - 3$$

$$4x - 2x \geq -3 - 7$$

$$2x \geq -10$$

$$\frac{2x}{2} \geq \frac{-10}{2}$$

$$x \geq -5$$



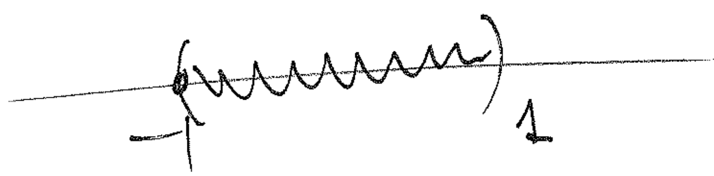
The solution set is $[-5, \infty)$

Ex $-3 < 2x-1 < 1$

$$-3+1 < 2x-1+1 < 1+1$$

$$\frac{-2}{2} < \frac{2x}{2} < \frac{2}{2}$$

$$-1 < x < 1$$



The solution set is $(-1, 1)$

$$(8x^3+2x^2-x+1)-(2x^3+x^2+1)$$

$$= 8x^3+2x^2-x+1-2x^3-x^2-1$$

$$= 6x^3+x^2-x$$

Ex $(2x+3)(x^2-x+1)$

$$= 2x x^2+2x(-x)+2x(1)+3x^2+3(-x)+3(1)$$

$$= 2x^3-2x^2+2x+3x^2-3x+3$$

$$= \boxed{2x^3+x^2-x+3}$$

Special formulas

$$\left.\begin{aligned}
(x+a)^2 &= x^2+2xa+a^2 \\
(x-a)^2 &= x^2-2ax+a^2 \\
(x-a)(x+a) &= x^2-a^2
\end{aligned}\right\} \text{Memorize}$$

$$(x+a)^3 = x^3+3a^2x+3x^2a+a^3$$

$$x^3-a^3 = (x-a)\left(x^2+ax+a^2\right)$$

$$x^3+a^3 = (x+a)\left(x^2-ax+a^2\right)$$

$$(2x+1)^2 = (2x)^2 + 2(2x)(1) + 1^2$$
$$= 4x^2 + 4x + 1$$

$$(x-1)^3 = (x-1)(x-1)^2$$
$$= (x-1)(x^2-2x+1)$$
$$= x^3 - 2x^2 + x - x^2 + 2x - 1$$
$$= x^3 - 3x^2 + 3x - 1$$

Long Division

$$\frac{x^2+2x+3}{x-1} = x+3+\frac{6}{x-1}$$

$$
\begin{array}{r l}
 & x+3 \\
\cline{2-2}
x-1 & \big)\, x^2+2x+3 \\
 & -x^2+x \\
\cline{2-2}
 & \qquad 3x+3 \\
 & \qquad -3x+3 \\
\cline{2-2}
 & \qquad\qquad 6
\end{array}
$$

Ex

$3x^3+4x^2+x+7$ Divided by x^2+1

$$\cancel{3x^3+4x^2+}\,x+7$$

$$
\begin{array}{r|l}
 & 3x+4 \\
\hline
x^2+1 & 3x^3+4x^2+x+7 \\
 & -3x^3 \qquad -3x \\
\hline
 & \qquad 4x^2-2x+7 \\
 & \qquad -4x^2 \qquad -4 \\
\hline
 & \qquad\qquad \boxed{-2x+3}
\end{array}
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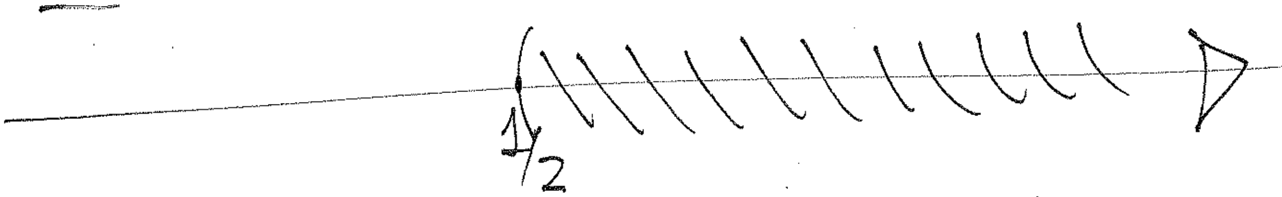

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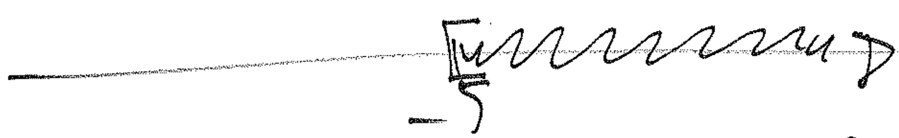

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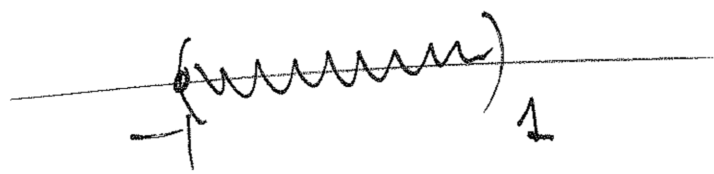

The solution set is $(-1, 1)$

Ex $(4x-1)^{-1} > 0$

$$\frac{1}{4x-1} > 0$$

$$4x-1 > 0$$

$$4x > \cancel{4}\,1$$

$$x > \frac{1}{4}$$

The solution set is

$$\left(\frac{1}{4}, \infty\right)$$

A3 A polynomial is a finite sum of terms of the type

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

$a_n, a_{n-1}, \ldots, a_1, a_0$ are the coefficients

a_n is the leading coefficient.

Ex $8x^3 + 4x^2 - 3x + 1$ is a polynomial of degree 3

$8x^2 - 3x^{-1} + 2$ is not a polynomial

$$(8x^3+2x^2-x+1)-(2x^3+x^2+1)$$

$$= 8x^3+2x^2-x+1-2x^3-x^2-1$$

$$= 6x^3+x^2-x$$

<u>Ex</u> $(2x+3)(x^2-x+1)$

$$= 2x\,x^2+2x(-x)+2x(1)+3x^2+3(-x)+3(1)$$

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$$= \boxed{2x^3+x^2-x+3}$$

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$$\left.\begin{aligned}
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$$x^3+a^3 = (x+a)\,(x^2-ax+a^2)$$

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Long Division

$$\frac{x^2+2x+3}{x-1} = x+3+\frac{6}{x-1}$$

$$
\begin{array}{r}
x+3 \\
x-1\ \big)\ \overline{x^2+2x+3} \\
-x^2+x \\
\hline
3x+3 \\
-3x+3 \\
\hline
6
\end{array}
$$

Ex

$3x^3+4x^2+x+7$ Divided by x^2+1

$$\cancel{3x^3+4x^2}+x+7$$

$$\begin{array}{r l}
 & \quad 3x+4 \\
x^2+1 & \overline{)\,3x^3+4x^2+x+7} \\
 & \underline{-3x^3 \qquad\quad -3x} \\
 & \qquad\quad 4x^2-2x+7 \\
 & \qquad\quad \underline{-4x^2 \qquad\ -4} \\
 & \qquad\qquad\quad \boxed{-2x+3}
\end{array}$$

$$\boxed{\frac{3x^3+4x^2+x+7}{x^2+1} = 3x+4+\frac{-2x+3}{x^2+1}}$$

Ex Factor completely

$$2x^2 - x - 6$$

$ac = -12$ $\quad -4, 3$

$b = -1$

$$\underbrace{2x^2 - 4x} + \underbrace{3x - 6}$$

$$2x(x-2) + 3(x-2)$$

$$\boxed{(x-2)(2x+3)}$$

$$\cancel{\frac{x^2-2x+1}{x^3+x} \cdot \frac{4x^2+4}{x^2+x-2}}$$

$$\frac{12}{x^2+x} \cdot \frac{x^3+1}{4x-2}$$

$$\frac{12\cdot(x^3+1)}{(x^2+x)(4x-2)} = \frac{\overset{6}{\cancel{12}}\,\cancel{(x+1)}(x^2-x+1)}{x\,\cancel{(x+1)}\,\cancel{2}(2x-1)}$$

$$= \frac{6(x^2-x+1)}{x(2x-1)} = \boxed{\frac{6x^2-6x+6}{2x^2-x}}$$

$$x^4 - 1 = (x^2)^2 - 1^2$$

$$= (x^2 - 1)(x^2 + 1)$$

$$= (x^2 - 1^2)(x^2 + 1)$$

$$= \boxed{(x-1)(x+1)(x^2+1)}$$

Ex

$$x^2 + 9x + 8$$

$$(x+1)(x+8)$$

Ex

$$2z^2 + 5z + 3$$

$$2z^2 + 2z + 3z + 3$$

$$2z(z+1) + 3(z+1)$$

$$\boxed{(z+1)(2z+3)}$$

simplify

$$\frac{x^2+4x+4}{x^2+3x+2} = \frac{(x+2)\cancel{(x+2)}}{\cancel{(x+2)}(x+1)}$$

$$= \frac{x+2}{x+1}$$

$$\frac{x^2-2x+1}{x^3+x} \cdot \frac{4x^2+4}{x^2+x-2} = \frac{(x^2-2x+1)(4x^2+4)}{(x^3+x)(x^2+x-2)}$$

$$= \frac{(x-1)\cancel{(x-1)}\,4\,\cancel{(x^2+1)}}{x\,\cancel{(x^2+1)}\,(x+2)\cancel{(x-1)}} = \frac{4(x-1)}{x(x+2)}$$

$$= \boxed{\frac{4x-4}{x^2+2x}}$$