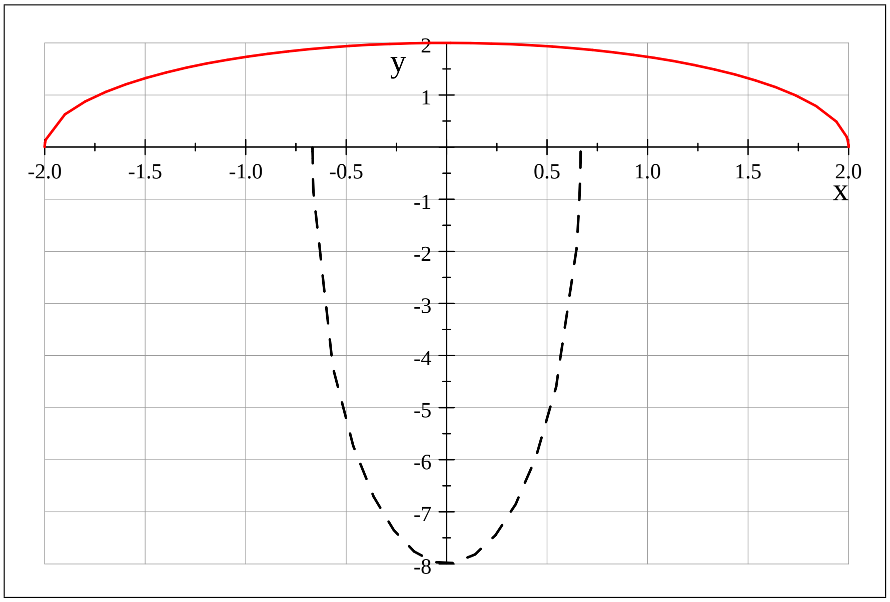
Graph transformations

Consider the graph of the function $f(x)$ in red and the graph of

$$F(x) = -4f(3x)$$

in black

$$\begin{aligned} f(x) &= \sqrt{4-x^2} \\ F(x) &= -4\sqrt{4-9x^2} \end{aligned}$$



We wish to understand what the point $(-2, 0)$ will correspond to on the graph of F. Since the new point must be an x intercept for the graph of F, then

$$\begin{aligned} F(x) &= 0 \Rightarrow -4f(3x) = 0 \\ &\Rightarrow f(3x) = 0 \\ &\Rightarrow 3x = -2 \text{ since } x \text{ must be negative} \\ &\Rightarrow x = -2/3. \end{aligned}$$

Thus $(-2, 0)$ corresponds to $(-2/3, 0)$. Similarly, to see what the point $(2, 0)$ must correspond to on the graph of F we compute

$$F(x) = 0 \Rightarrow -4f(3x) = 0$$

$$\begin{aligned} &\Rightarrow \quad f(3x) = 0 \\ &\Rightarrow \quad 3x = 2 \text{ since } x \text{ must be positive} \\ &\Rightarrow \quad x = 2/3. \end{aligned}$$

Thus $(2, 0)$ corresponds to $(2/3, 0)$. Finally, we compute what the point $(0, 1)$ must correspond to on the graph of F. One thing that is clear so far is that it must correspond to some extremum. But the extremum value for F is actually -8. Thus we solve the equation

$$\begin{aligned} F(x) &= -8 \Rightarrow -4f(3x) = -8 \\ &\Rightarrow f(3x) = 2 \\ &\Rightarrow 3x = 0 \\ &\Rightarrow x = 0. \end{aligned}$$

$(0, 1)$ corresponds to $(0, -8)$.