5.4 Properties of Trig functions

Reciprocal Identities

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

Identities for negatives

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

Pythagorean Identity

$$\sin^2 x + \cos^2 x = 1$$

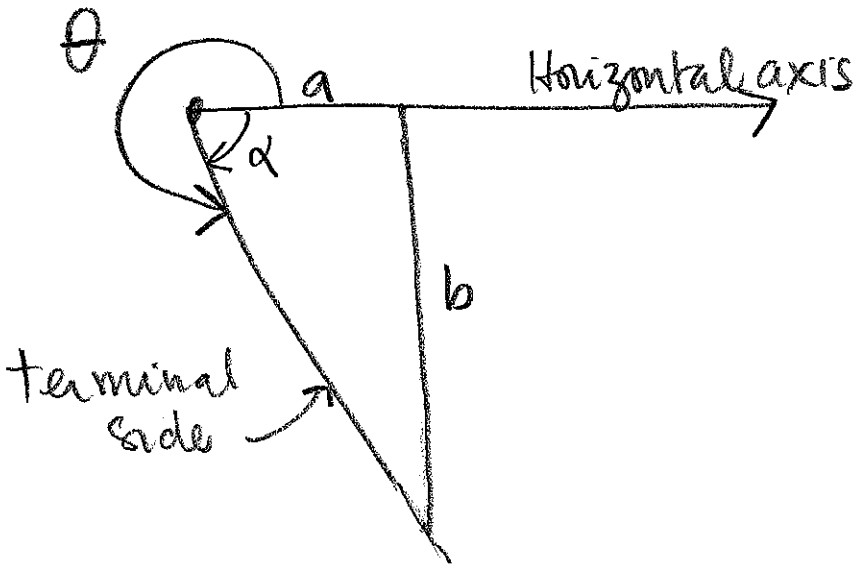
Ex Use the basic identities to find the values of the other five trig functions

$\sin x = -\frac{2}{3}$ and $\tan x > 0$

Solution (The student should work this problem out)

Reference Triangle and reference angle

Given an angle θ, we form the reference triangle as follow.



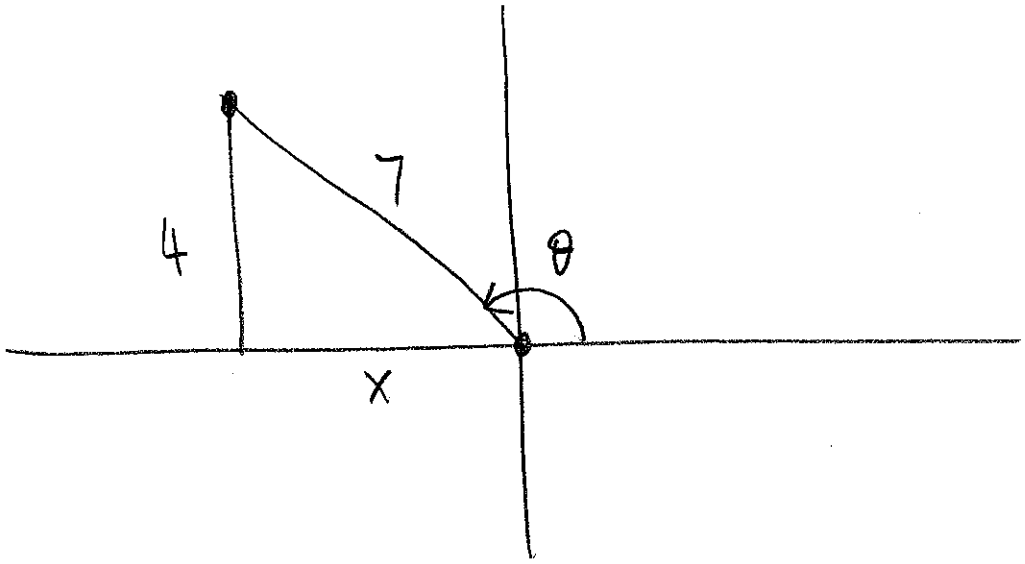
- The Reference angle α is the acute positive angle taken between the terminal side and the horizontal axis. We have

$$\sin\theta = \frac{Opp}{Hyp}, \quad a = Adj, \quad b = Opp$$

$$\cos\theta = \frac{Adj}{Hyp}$$

$$\tan\theta = \frac{Opp}{Adj}$$

Ex If $\sin\theta = 4/7$, $\cos\theta < 0$, find the values of each of the other five trig functions.



$\cos\theta = \frac{x}{7}$ but $x^2 + 16 = 49 \Rightarrow x = -\sqrt{33}$

thus $\cos\theta = -\frac{\sqrt{33}}{7}$

$\tan\theta = \frac{4}{7} \times \frac{7}{\sqrt{33}} = -\frac{4}{\sqrt{33}}$

$\cot\theta = -\frac{\sqrt{33}}{4}$

$\sec\theta = -\frac{7}{\sqrt{33}}$

$\csc\theta = \frac{7}{4}$

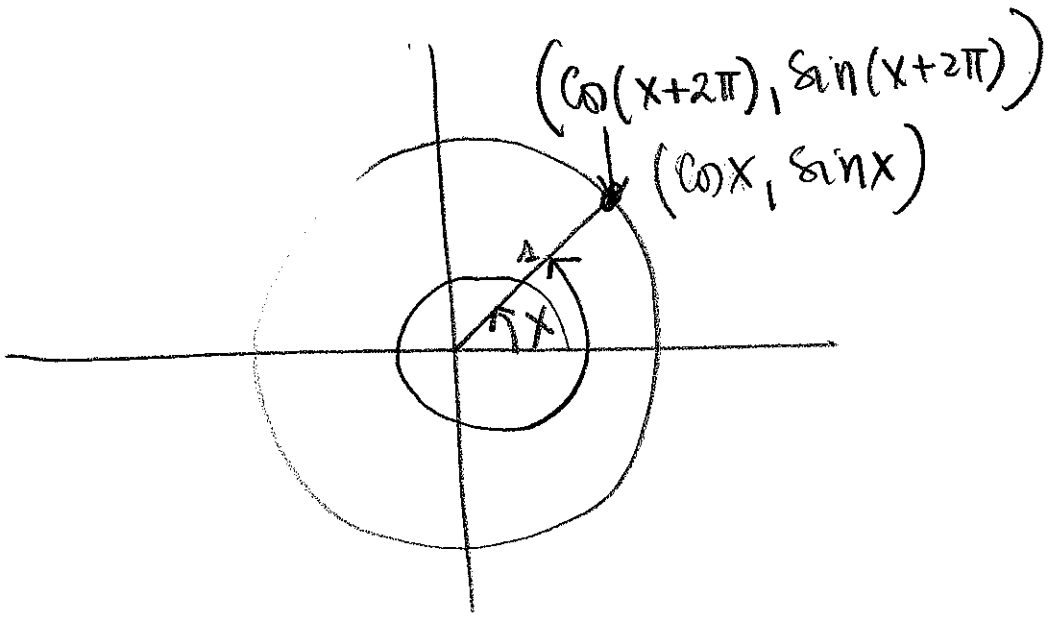
Periodic functions

A function f is periodic if there exists a positive real number p such that

$$f(x+p) = f(x)$$

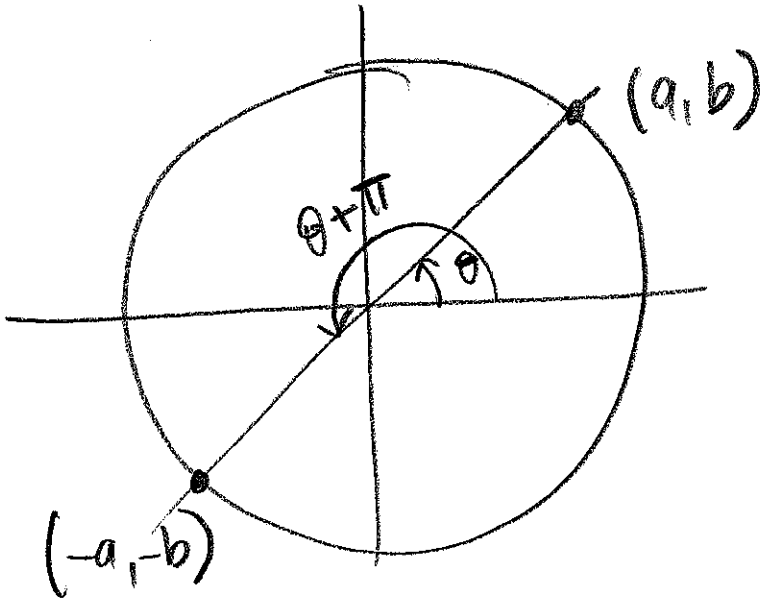
The smallest such positive p if it exists is called the <u>period</u> of f

Ex



Sine and cosine repeat every 2π units, and 2π is the smallest such positive number. Thus both sine and cosine have for period 2π

Claim Tangent is a periodic function but its period is π NOT 2π!



$$\tan(\theta) = \frac{b}{a}$$

$$\tan(\theta+\pi) = \frac{-b}{-a} = \frac{b}{a}$$

Thus $\tan(\theta+\pi) = \tan\theta$.

Geometrically, we see that π is the smallest possitive value such that $\tan(\theta+\pi) = \tan\theta$.

thus the period of tangent is π.

Worksheet

1. Find all the other trig functions if

a) $\sin\theta = \frac{3}{5}$ and $\cos\theta < 0$

b) $\sec\theta = 10$ and $\tan\theta < 0$

2. Without a calculator find the smallest positive θ in degree and radian for which

a) $\cos\theta = -1/2$

b) $\sin\theta = -\frac{\pi}{4}$

3.) Find all functions of the form $f(x) = ax^2 + bx + c$ that are periodic