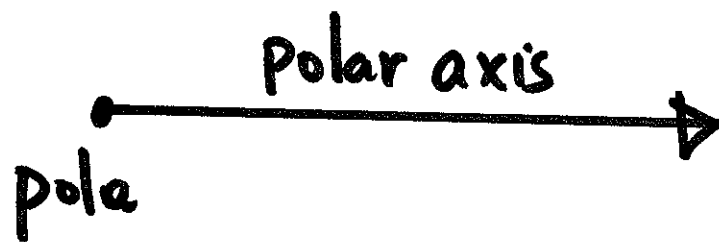


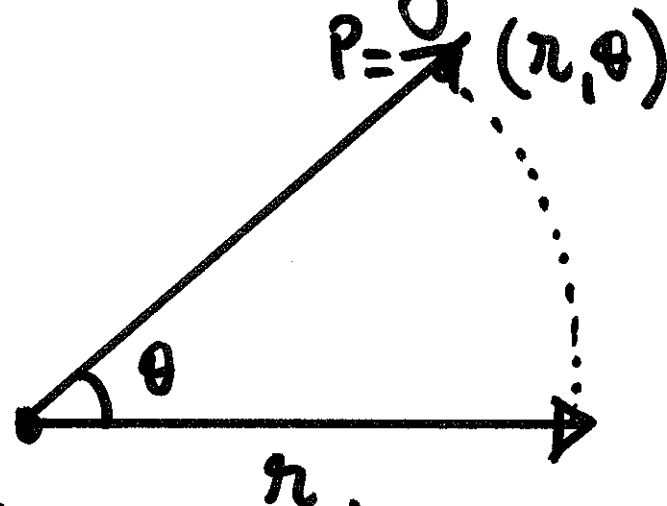
7.4 Polar Coordinates and Graphs

To form a polar coordinate system, need

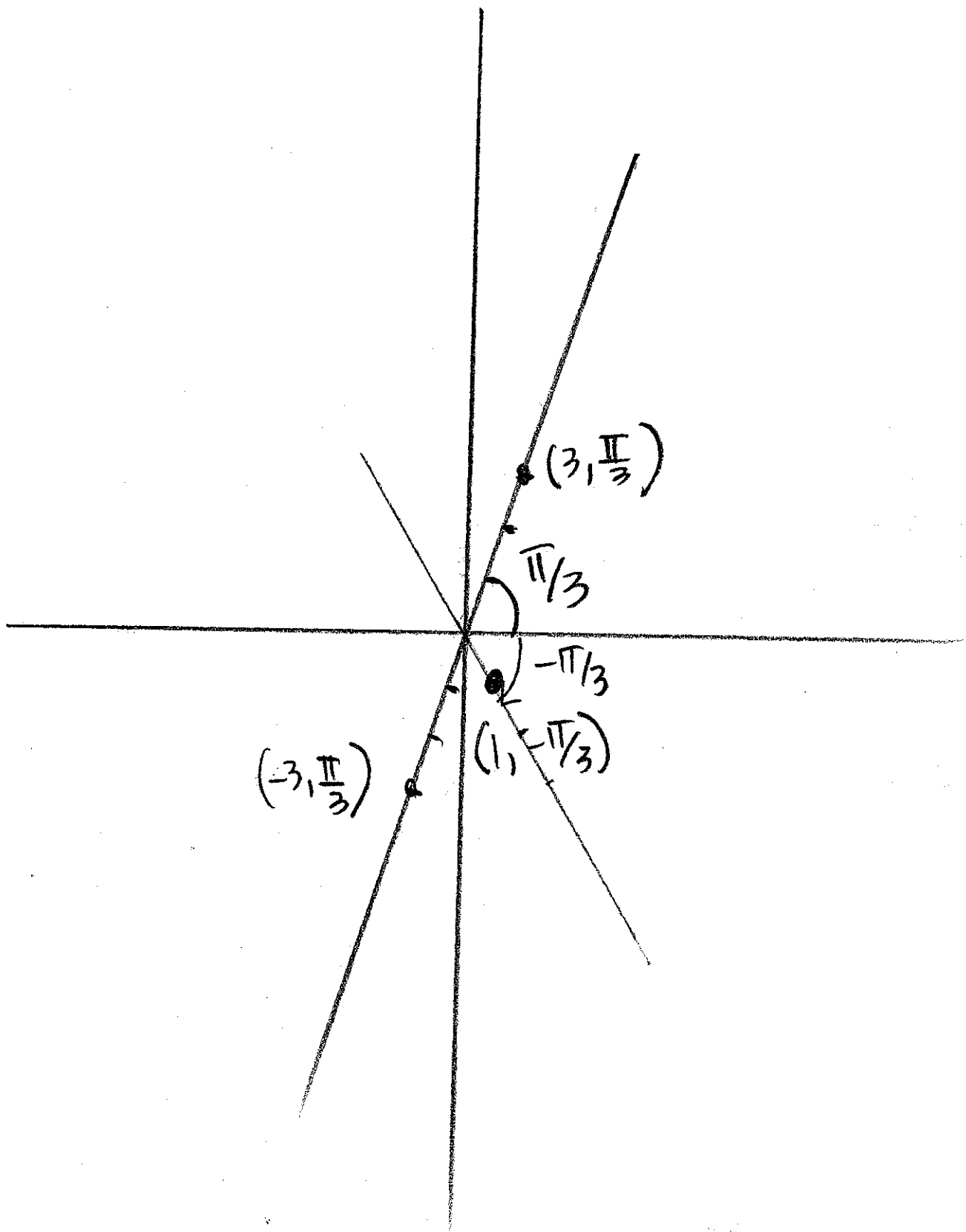
- A pole
- A polar axis

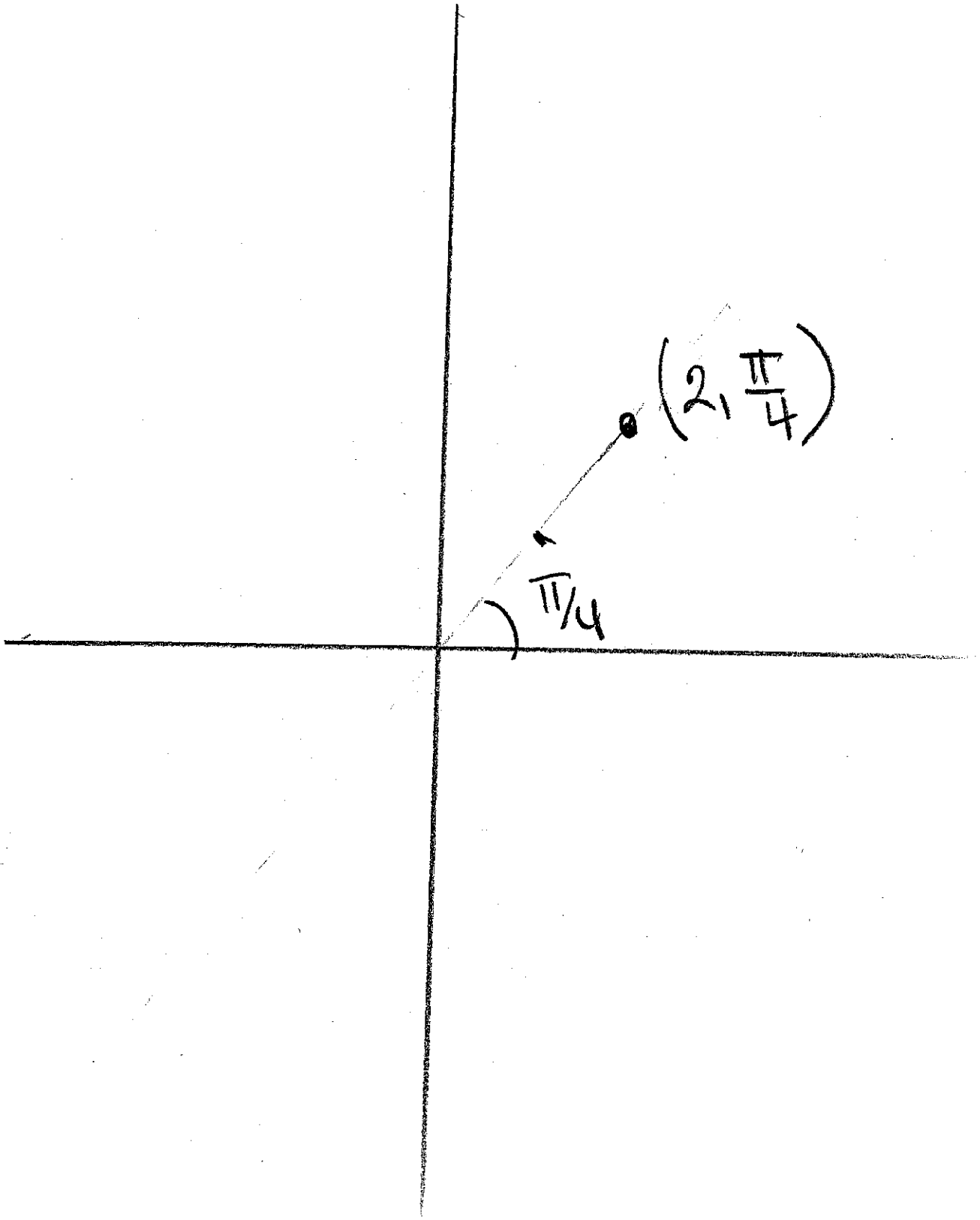


We rotate the polar axis by an angle θ

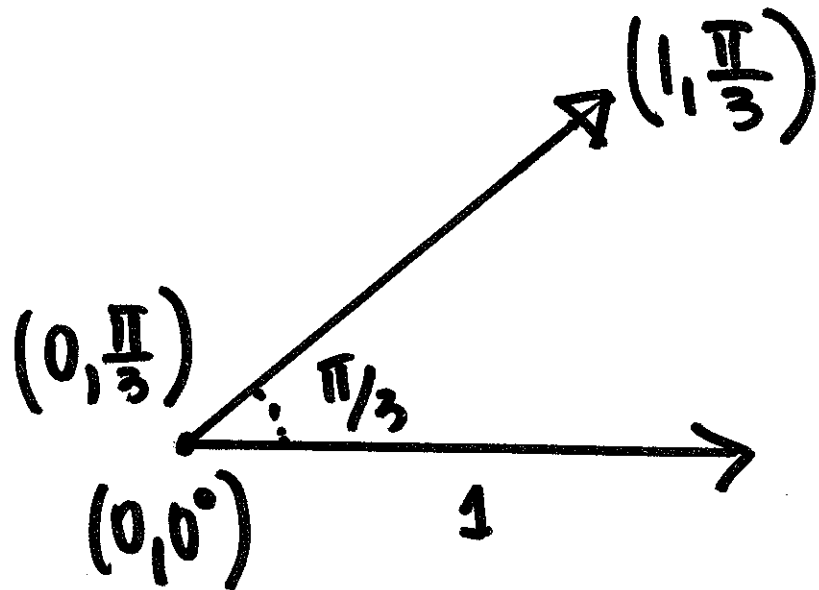


- If $\theta > 0$, rotate counterclockwise
- If $\theta < 0$, rotate clockwise
- If $r < 0$, move in opposite direction of the rotated polar axis.





Example Plot $(0, 0^\circ)$, $(0, \pi/3)$,
 $(1, \pi/3)$



Ex Plot $(3, \pi/3)$, $(-3, \pi/3)$
 $(1, -\pi/3)$, $(2, \pi/4)$

Polar-rectangular Relationships

$$r^2 = x^2 + y^2$$

Rectangular coord : (x, y)

Polar coord : (r, θ)

$$r^2 = x^2 + y^2$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\tan \theta = \frac{y}{x}$$

The sign of x, y determine the quadrant for θ .

Usually we choose θ so that

$-\pi < \theta \leq \pi$, unless stated otherwise.

Ex Convert $(-4, \pi/4)$ to rectangular coordinates.

$$r = -4, \theta = \frac{\pi}{4}$$

$$x = -4 \cos \frac{\pi}{4} = -4 \frac{\sqrt{2}}{2} = -2\sqrt{2}$$

$$y = -4 \sin \frac{\pi}{4} = -2\sqrt{2}$$

$$(-2\sqrt{2}, -2\sqrt{2})$$

Ex Convert $(1, 3)$ to polar coordinates.

$$x = 1$$

$$y = 3$$

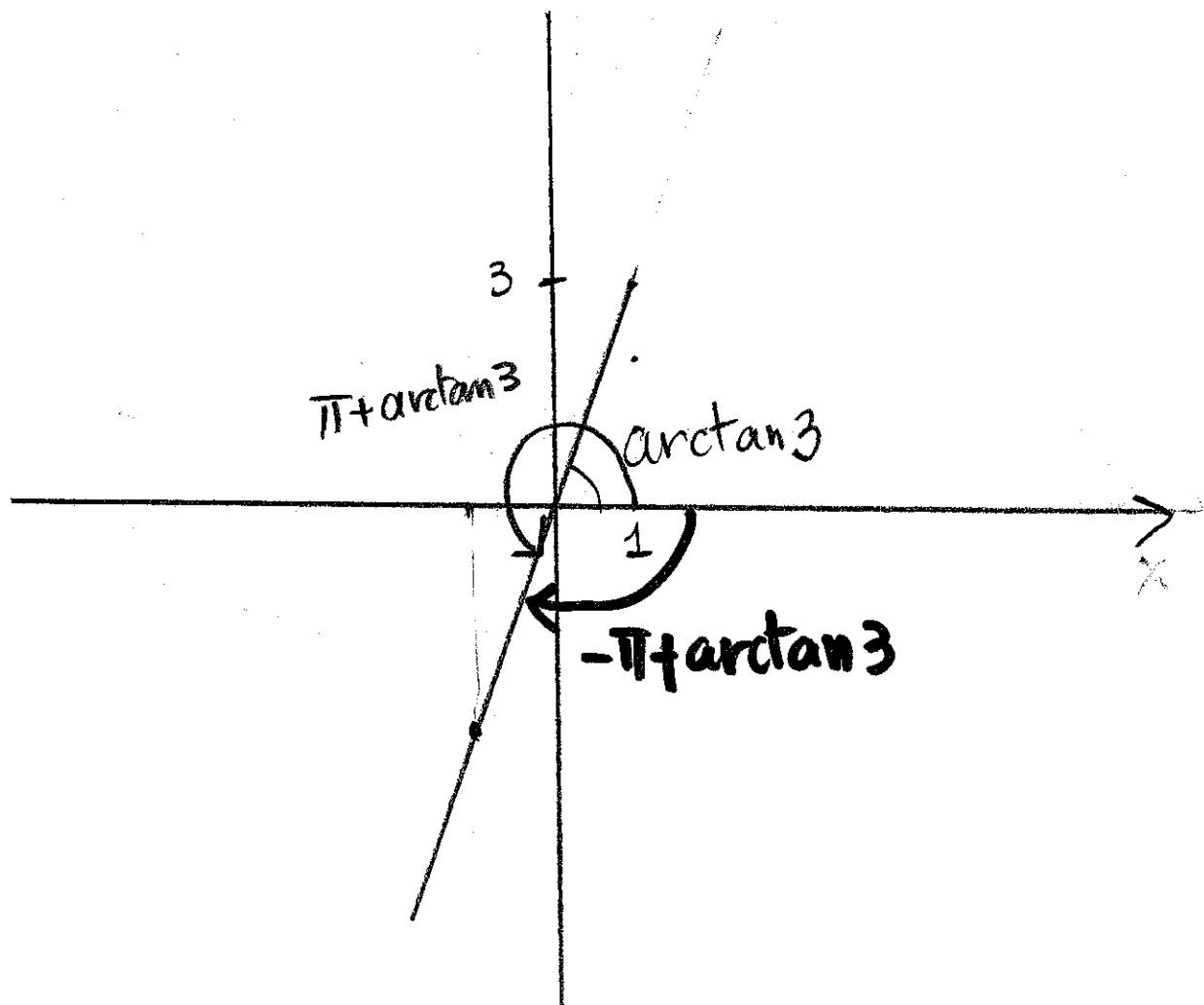
$$r^2 = x^2 + y^2 = 1 + 9 = 10.$$

$$r = \sqrt{10}$$

$$\frac{y}{x} = 3 \Rightarrow \tan \theta = 3 \Rightarrow \theta = \arctan 3$$

$$(\sqrt{10}, \arctan 3)$$

What if we pick $r = -\sqrt{10}$?



have

$$\left(-\sqrt{10}, \pi + \arctan 3\right)$$

or

$$\left(-\sqrt{10}, -\pi + \arctan 3\right)$$

Ex Change $x^2 + y^2 - 4y = 0$ to polar form.

Put $r^2 = x^2 + y^2$, $y = r \sin \theta$

$x^2 + y^2 - 4y = 0$ becomes

$$r^2 - 4r \sin \theta = 0$$

$$r(r - 4 \sin \theta) = 0$$

$$r = 0 \text{ or } r = 4 \sin \theta$$

Ex Change $r = -3 \cos \theta$ to Rectangular form

~~$x^2 + y^2 = -3x$~~ $r^2 = -3r \cos \theta$

$$x^2 + y^2 = -3x$$

$$x^2 + y^2 + 3x = 0$$