Section 6.2 Sum, Difference and cofunction identities

1. **Sum and differences identities**

$$\begin{aligned}
\cos(x+y) &= \cos x\cos y - \sin x\sin y \\
\cos(x-y) &= \cos x\cos y + \sin x\sin y \\
\sin(x+y) &= \sin x\cos y + \cos x\sin y \\
\sin(x-y) &= \sin x\cos y - \cos x\sin y \\
\tan(x+y) &= \frac{\tan x + \tan y}{1-\tan x\tan y} \\
\tan(x-y) &= \frac{\tan x - \tan y}{1+\tan x\tan y}
\end{aligned}$$

2. **Cofunctions identities**

$$\begin{aligned}
\cos\left(\frac{\pi}{2}-x\right) &= \sin x \\
\sin\left(\frac{\pi}{2}-x\right) &= \cos x \\
\tan\left(\frac{\pi}{2}-x\right) &= \cot(x)
\end{aligned}$$

3. **Double angle identities**

$$\begin{aligned}
\sin 2x &= 2\sin x\cos x \\
\cos 2x &= \cos^2 x - \sin^2 x \\
\cos 2x &= 1 - 2\sin^2 x \\
\cos 2x &= 2\cos^2 x - 1 \\
\tan(2x) &= \frac{2\tan(x)}{1-\tan^2(x)}
\end{aligned}$$

4. Half angle identities

$$\begin{aligned}
\sin\frac{x}{2} &= \pm\sqrt{\frac{1-\cos x}{2}} \\
\cos\frac{x}{2} &= \pm\sqrt{\frac{1+\cos x}{2}} \\
\tan\frac{x}{2} &= \frac{\pm\sqrt{\frac{1-\cos x}{2}}}{\pm\sqrt{\frac{1+\cos x}{2}}} \\
&= \pm\sqrt{\frac{1-\cos x}{1+\cos x}}
\end{aligned}$$

Exercises for section 6.3

5. Prove that
$$\cos 4x = \cos^4 x - 6\cos^2 x \sin^2 x + \sin^4 x$$

6. Show that
$$\sin\left(\frac{\pi}{12}\right) = \sqrt{\frac{2-\sqrt{3}}{2}}$$

7. Without a calculator, compute
$$\cos\left(\frac{13}{12}\pi\right)$$
Hint: $\frac{\pi}{3} + \frac{\pi}{2} + \frac{\pi}{4} = \frac{13\pi}{12}$.

8. Verify the identity
$$1 - \cos 2x = \tan x \sin 2x$$

9. Prove the following identity
$$\cos^4\theta = \frac{1}{4} + \frac{\cos 2\theta}{2} + \frac{\cos^2 2\theta}{4}$$

10. Use the half-angle formulas to find
$$\cos\left(105\,\text{deg}\right)$$

11. Without a calculator evaluate the following
$$\cos\left(\frac{1}{2}\arccos\left(-\frac{3}{5}\right)\right)$$

12. Show that
$$\cos^2 x = \frac{\cos 2x + 1}{2}$$

13. Use question 8 to show that
$$\begin{aligned}\cos^6 x \;=\; & \frac{1}{8}\cos^6 x - \frac{3}{8}\cos^4 x \sin^2 x + \frac{3}{8}\cos^4 x + \frac{3}{8}\cos^2 x \sin^4 x - \frac{3}{4}\cos^2 x \sin^2 x + \\ & \frac{3}{8}\cos^2 x - \frac{1}{8}\sin^6 x + \frac{3}{8}\sin^4 x - \frac{3}{8}\sin^2 x + \frac{1}{8}.\end{aligned}$$

14. Show that
$$\tan\frac{\alpha}{2} = \frac{\sec\alpha}{\sec\alpha\csc\alpha + \csc\alpha}$$