

# **Section 6.2 Sum, Difference, and Cofunction**

## THEOREM

### Sum and Difference Formulas for Cosines

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

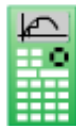
$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

Find the exact value of  $\cos \frac{7\pi}{12}$ .

Find the exact value of  $\cos 15^\circ$ .

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$



## Seeing the Concept

Graph  $Y_1 = \cos\left(\frac{\pi}{2} - x\right)$  and  $Y_2 = \sin x$  on the same screen. Does this demonstrate the result 3(a)? How would you demonstrate the result 3(b)?

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## THEOREM

### Sum and Difference Formulas for Sines

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Find the exact value of  $\sin \frac{19\pi}{12}$ .

Find the exact value of  $\cos 40^\circ \cos 80^\circ - \sin 40^\circ \sin 80^\circ$ .

If it is known that  $\sin \alpha = \frac{3}{5}$ ,  $\frac{\pi}{2} < \alpha < \pi$ , and that  $\sin \beta = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$ ,

$\frac{3\pi}{2} < \beta < 2\pi$ , find the exact value of

- (a)  $\cos \alpha$       (b)  $\cos \beta$       (c)  $\cos(\alpha + \beta)$

Establish the identity:

$$\cos\left(\frac{\pi}{2} + \theta\right) = -\sin \theta$$

## THEOREM

### Sum and Difference Formulas for Tangents

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

Prove the identity:  $\tan(2\pi - \theta) = -\tan \theta$

Prove the identity:

$$\tan\left(\frac{\pi}{4} + \theta\right) = \cot\left(\frac{\pi}{4} - \theta\right)$$

Find the exact value of:  $\cos\left(\sin^{-1}\frac{2}{3} + \tan^{-1}\left(-\frac{3}{4}\right)\right) = \cos(\alpha + \beta)$

Solve the equation:  $\sin \theta + \cos \theta = 1$ ,  $0 \leq \theta < 2\pi$

## SUMMARY Sum and Difference Formulas

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$