Section 6.2 Sum, Difference, and Cofunction

THEOREM

Sum and Difference Formulas for Cosines

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

Find the exact value of $\cos\frac{7\pi}{12}$.

Find the exact value of $\cos 15°$.

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

Seeing the Concept

Graph $Y_1 = \cos\left(\frac{\pi}{2} - x\right)$ and $Y_2 = \sin x$ on the same screen. Does this demonstrate the result 3(a)? How would you demonstrate the result 3(b)?

THEOREM

Sum and Difference Formulas for Sines

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

Find the exact value of $\sin \frac{19\pi}{12}$.

Find the exact value of $\cos 40° \cos 80° - \sin 40° \sin 80°$.

If it is known that $\sin\alpha = \frac{3}{5},\ \frac{\pi}{2} < \alpha < \pi,$ and that $\sin\beta = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5},$

$\frac{3\pi}{2} < \beta < 2\pi,$ find the exact value of

(a) $\cos\alpha$ (b) $\cos\beta$ (c) $\cos(\alpha + \beta)$

Establish the identity:

$$\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$$

THEOREM

Sum and Difference Formulas for Tangents

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Prove the identity: $\tan\left(2\pi-\theta\right)=-\tan\theta$

Prove the identity:

$$\tan\left(\frac{\pi}{4}+\theta\right)=\cot\left(\frac{\pi}{4}-\theta\right)$$

Find the exact value of: $\cos\left(\sin^{-1}\frac{2}{3}+\tan^{-1}\left(-\frac{3}{4}\right)\right)=\cos(\alpha+\beta)$

Solve the equation: $\sin\theta + \cos\theta = 1, \quad 0 \leq \theta < 2\pi$

SUMMARY Sum and Difference Formulas

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$