6.2 Sum, Difference and Cofunction identities

Question Is $\sin(A+B) = \sin A + \sin B$?

Hmk: P582 #15, 21, 33, 37, 41, 49, 53,

Sum

$$\sin(x+y) = \sin x \cos y + \sin y \cos x$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

Difference

$$\sin(x-y) = \sin x \cos y - \sin y \cos x$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Ex Simplify $\cos(x-\pi)$

$$\cos x \cos\pi + \sin x \sin\pi = -\cos x$$

Ex Simplify $\sin(x-\pi)$

$$\sin x \cos(\pi) - \sin\pi \cos x = -\sin x$$

Ex Find the exact value of $\tan 75°$

$1/2 \cdot \frac{2}{\sqrt{3}}$

$$\tan(75) = \tan(45+30)$$

$$= \frac{\tan 45 + \tan 30}{1 - \tan 45 \cdot \tan 30} = \frac{1 + 1/\sqrt{3}}{1 - 1/\sqrt{3}}$$

$$= \boxed{\frac{\sqrt{3}+1}{\sqrt{3}-1}}$$

Ex Find the exact value of

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$$
$$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$$
$$= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

Ex Write $\cos 3x \cos 2x - \sin 3x \sin 2x$ as a single cosine

$\cos(5x)$.

Ex Find $\cos(105°)$

$180 \to \pi$
$105 \to x$
$x = \frac{105\pi}{180}$

$$\cos(105) = \cos\left(\frac{7\pi}{12}\right)$$
$$= \cos\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right)$$
$$= \cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$$
$$= \frac{\sqrt{2}}{2}\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{2}-\sqrt{6}}{4}$$

Ex If $\tan(A+B) = 3$, $\tan B = \frac{1}{2}$ find $\tan A$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$3 = \frac{\tan A + 1/2}{1 - \tan A/2} = \frac{2\tan A + 1}{2 - \tan A}$$

$$2\tan A + 1 = 6 - 3\tan A$$

$$5\tan A = 5 \Rightarrow \boxed{\tan A = 1}$$

Homework-Quiz

1. Show that

$$\frac{\cos x}{1-\sin^2 x} = \sec x$$

2. Show that

$$\sin^4 w - \cos^4 w = 1 - 2\cos^2 w$$

6.3 Double-angle and Half angle identity

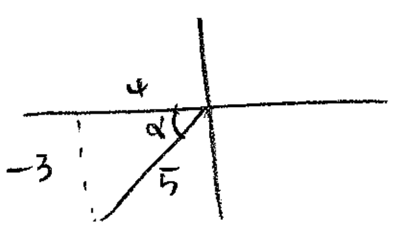
Hmk Pg 593
#7, 9, 17, 21
23, 29, 33, 35, 61
Quiz Monday

1. $\sin 2x = 2\sin x\cos x$
2. $\cos 2x = \cos^2 x - \sin^2 x$
3. $\cos 2x = 2\cos^2 x - 1$
4. $\cos 2x = 1 - 2\sin^2 x$
5. $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

Ex Verify that $\cos 2x = \dfrac{1-\tan^2 x}{1+\tan^2 x}$

$$\frac{1-\tan^2 x}{1+\tan^2 x} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{\cos^2 x + \sin^2 x} = \cos^2 x - \sin^2 x = \cos 2x$$

Ex Find the exact value of $\sin 2x$, $\cos 2x$ if $\tan x = -3/4$ and x is in the IV quadrant.



$\sin x = -3/5$, $\cos x = 4/5$

Now, $\sin 2x = 2\sin x\cos x = 2\cdot\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = -\frac{24}{25}$

$\cos 2x = \cos^2 x - \sin^2 x = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

Half-angle identity

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

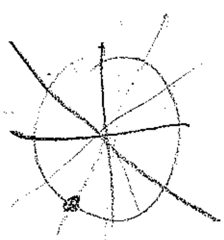
$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan x = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

The sign is determined by the quadrant in which $x/2$ lies.

<u>Ex</u> Find the exact value of $\cos(5\pi/8)$

$$\cos\left(\frac{5\pi}{8}\right) = \cos\left(\frac{5\pi/4}{2}\right) \leftarrow \text{Half angle}$$



$$= \pm\sqrt{\frac{1+\cos\frac{5\pi}{4}}{2}}$$

$$= -\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$$

$$= -\sqrt{\frac{2-\sqrt{2}}{4}}$$

$$= -\frac{\sqrt{2-\sqrt{2}}}{2}$$

<u>Ex</u> Verify that

$$\sin^2\left(\frac{x}{2}\right) = \frac{\tan x - \sin x}{2\tan x}$$

Since

$$LHS = \sin^2\left(\frac{x}{2}\right) = \frac{1-\cos x}{2}$$

and

$$RHS = \frac{1}{2} - \sin x \cdot \frac{\cos x}{2\sin x}$$

$$= \frac{1}{2} - \frac{1}{2}\cos x$$

$$= \frac{1-\cos x}{2}$$

The identity is verified.