5-4 Properties of Trig Functions

Homework Page 515 # 27,29,31,33,35,41,43,47,65

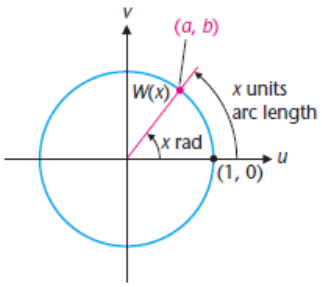
1. Trig functions

Let x be a real number and let (a, b) be the coordinates of the circular point $W(x)$ that lies on the terminal side of the angle with radian measure x. Then:

$$\sin x = b \qquad \csc x = \frac{1}{b} \quad b \neq 0$$

$$\cos x = a \qquad \sec x = \frac{1}{a} \quad a \neq 0$$

$$\tan x = \frac{b}{a} \quad a \neq 0 \qquad \cot x = \frac{a}{b} \quad b \neq 0$$



2. **Basic Trig identities**

   (a) **Reciprocal identities**

$$\begin{aligned} \csc x &= \frac{1}{\sin x} \\ \sec x &= \frac{1}{\cos x} \\ \cot x &= \frac{1}{\tan x} \end{aligned}$$

   (b) **Quotient identities**

$$\begin{aligned} \tan x &= \frac{\sin x}{\cos x} \\ \cot x &= \frac{\cos x}{\sin x} \end{aligned}$$

   (c) **Identities for negatives**

$$\begin{aligned} \sin(-x) &= -\sin(x) \\ \cos(-x) &= \cos(x) \\ \tan(-x) &= -\tan x \end{aligned}$$

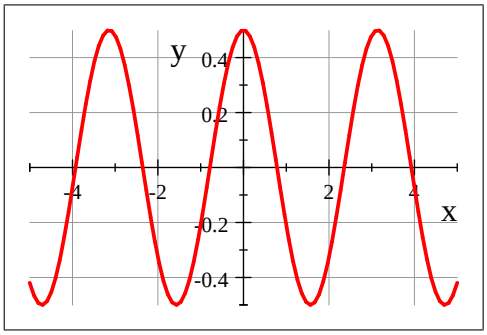
   (d) **Pythagorean identity**

$$\sin^2 x + \cos^2 x = 1$$

3. **Example** Use the basis identities to find the values of the other five trig functions given $\sin x = -\frac{1}{3}$ and $\tan x > 0$

4. **Example** If $\sin t = 2/7$ and $\cos t < 0$, find the values of each of the other five trig functions of θ

5. A function f is said to be periodic if there exists a positive real number p such that $f(x+p) = f(x)$ for all x in the domain of f. The smallest such positive p, if it exists, is called the fundamental period of f (or often just the period of f).$\cos x$



A periodic function

6. A function is odd if $f(-x) = -f(x)$ and a function is even if $f(-x) = f(x)$. An even function is symmetrical with respect to the y-axis, while an odd function is symmetrical with respect to the origin. For example check that

   (a) $y = \frac{\sin x}{x}$ is an even function

   (b) $y = \frac{\cos x}{x}$ is an odd function

Worksheet

1. Find the value of each of the other five trigonometric functions for an angle

   (a) $\sin t = 3/5$ and $\cos t < 0$

   (b) $\cos t = -\sqrt{5}/3$ and $\cot t > 0$

2. Determine each function is even odd, or neither.

   (a) $y = \frac{\sec x}{x}$

   (b) $y = x^3 \sin x$

3. If f and g are both even, then the function fg is odd. True or false

4. If f and g are each periodic with period p, then the function f/g is periodic.