4.3 Log functions

a By definition $y = \log_b x$ if and only if $b^y = x$

b The inverse function of $\log_b x$ is b^x

c

Let $f(x) = \log_b x$ be a logarithmic function, $b > 0, b \neq 1$. Then the graph of $f(x)$:

1. Is continuous on its domain $(0, \infty)$
2. Has no sharp corners
3. Passes through the point $(1, 0)$
4. Lies to the right of the y axis, which is a vertical asymptote
5. Is increasing as x increases if $b > 1$; is decreasing as x increases if $0 < b < 1$
6. Intersects any horizontal line exactly once, so is one-to-one

d

$$\begin{aligned}
\log_b 1 &= 0 \\
\log_b b &= 1 \\
\log_b b^x &= x \\
b^{\log_b x} &= x, x > 0 \\
\log_b MN &= \log_b M + \log_b N \\
\log_b \frac{M}{N} &= \log_b M - \log_b N \\
\log_b M^n &= n \log_b M
\end{aligned}$$

e $\log_e(x)$ is called the natural log of x and denoted $\ln x$.

Examples

1. Write in Exponential form

$$\begin{aligned}
\log_{10} 0.001 &= -3 \\
\log_2 \frac{1}{64} &= -6
\end{aligned}$$

2. Simplify the following expressions

$$\log_{25} 1$$
$$\log_{10} 10^5$$
$$\log_{1/5} \left(\frac{1}{25}\right)$$

3. Rewrite the expression in terms of $\log x$ and $\log y$

$$\log\left(\frac{x}{y}\right)$$
$$\log\left(x^4y^3\right)$$
$$\log\left(\frac{x^2}{\sqrt{y}}\right)$$

4. Rewrite the following as a single log

$$2\ln x + 5\ln y - \ln z$$

5. Find f^{-1}

$$f(x) = 2\log_2(x-5)$$

Solutions

1.

$$\begin{aligned}\log_{10} 0.001 &= -3\\ 10^{-3} &= 0.001\\ \log_2 \frac{1}{64} &= -6\\ 2^{-6} &= 1/64\end{aligned}$$

2.

$$\log_{25} 1 = 0$$
$$\log_{10}\left(10^5\right) = 5\log_{10} 10 = 5\times 1 = 5$$
$$\log_{1/5}\left(\frac{1}{25}\right) = \log_{1/5}\left(\left(\frac{1}{5}\right)^2\right) = 2\log_{1/5}(1/5) = 2$$

3.

$$\log\left(\frac{x}{y}\right) = \log x - \log y$$

$$\begin{aligned}&\log\left(x^4y^3\right) = \log\left(x^4\right) + \log\left(y^3\right)\\ =\ &4\log x + 3\log y\end{aligned}$$

$$\begin{aligned}\log\left(\frac{x^2}{\sqrt{y}}\right) &= \log\left(x^2\right) - \log\left(\sqrt{y}\right)\\ &= 2\log(x) - (1/2)\log y\end{aligned}$$

4.

$$
\begin{aligned}
2\ln x + 5\ln y - \ln z &= \ln\left(x^2\right) + \ln\left(y^5\right) - \ln z \\
&= \ln\left(x^2 y^5 z\right).
\end{aligned}
$$

5. Put

$$
\begin{aligned}
y &= 2\log_2(x-5) \\
y &= \log_2\left((x-5)^2\right) \\
2^y &= (x-5)^2 \\
x-5 &= \sqrt{2^y} \\
x &= \sqrt{2^y} + 5
\end{aligned}
$$

Thus $f^{-1}(x) = \sqrt{2^x} + 5$