

Section 4.1 Exponential Function

Definition 1

The equation $f(x) = b^x, b > 0, b \neq 1$ defines an exponential function. b is called the base of the exponential.

Theorem 2 *Properties of Graphs of exponential functions*

Let $f(x) = b^x$ be an exponential function, $b > 0, b \neq 1$. Then the graph of $f(x)$:

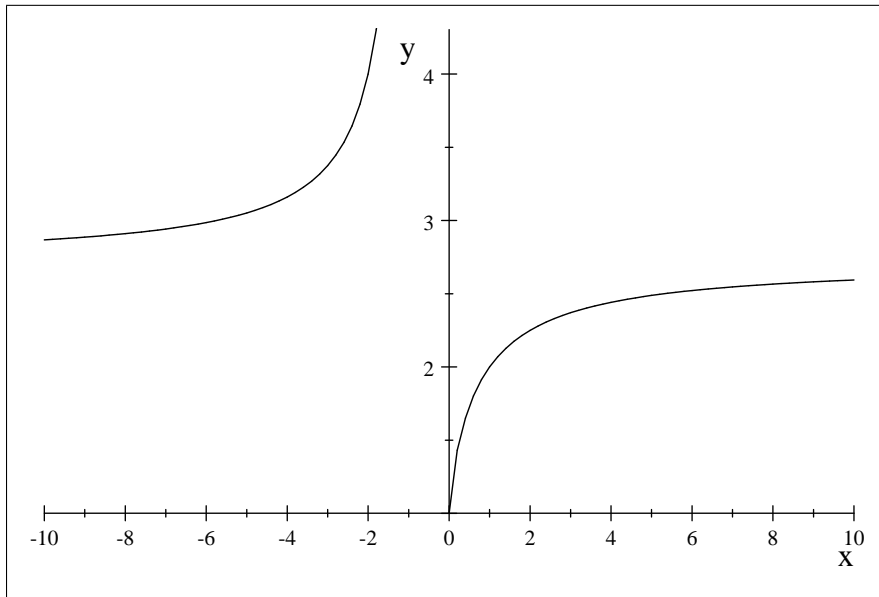
1. Is continuous for all real numbers
2. Has no sharp corners
3. Passes through the point $(0, 1)$
4. Lies above the x axis, which is a horizontal asymptote either as $x \rightarrow \infty$ or $x \rightarrow -\infty$, but not both
5. Increases as x increases if $b > 1$; decreases as x increases if $0 < b < 1$
6. Intersects any horizontal line at most once (that is, f is one-to-one)

Theorem 3 *Exponential properties*

1. $a^x a^y = a^{x+y}$
2. $(a^x)^y = a^{xy}$
3. $(ab)^x = a^x b^x$
4. $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
5. $\frac{a^x}{a^y} = a^{x-y}$
6. $a^x = a^y \Leftrightarrow x = y$
7. $a^x = b^x \Leftrightarrow a = b$

Definition 4 *By definition, the irrational number e called exponential is*

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$



graph of $(1 + \frac{1}{x})^x$

Definition 5

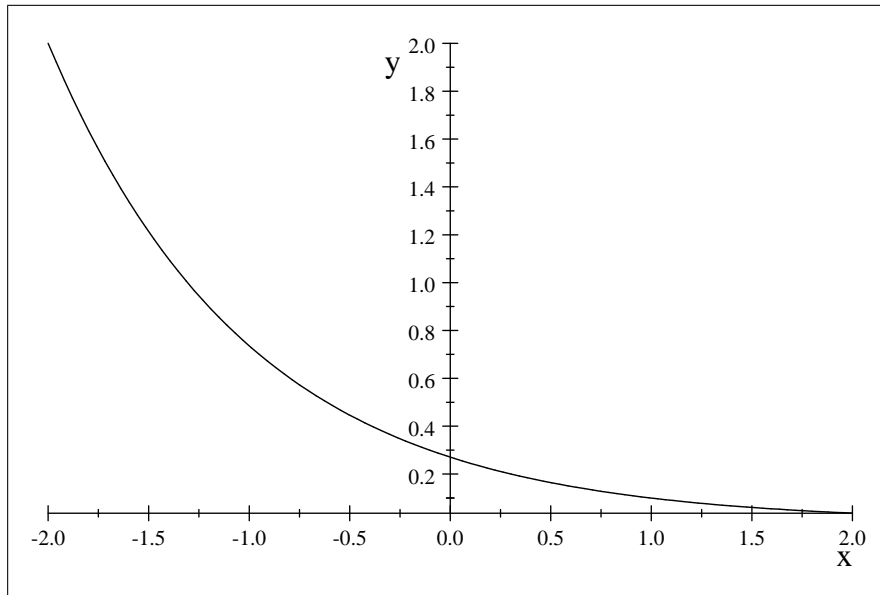
The function $f(x) = e^x$ is called the exponential function with base e

Example 6

1. Simplify the followings $\frac{e^{4-3x}}{e^{2-5x}}, 10^{3x-1}10^{4-x}$
2. Graph the function $g(x) = 2e^{-(x+2)}$ by using the appropriate transformations
3. Solve the equation $7^{x^2} = 7^{2x+3}$
4. Solve the equation $x^2e^x - 5xe^x = 0$
5. Find all numbers a such that $a^2 = a^{-2}$

Solution 7

1. Will be shown in class
- 2.



3. The exponents must be the same, thus,

$$\begin{aligned}x^2 &= 2x + 3 \\x^2 - 2x - 3 &= 0\end{aligned}$$

Solution is: 3, -1

- 4.

$$\begin{aligned}x^2 e^x - 5x e^x &= 0 \\e^x (x^2 - 5x) &= 0 \\e^x &= 0 \text{ or } x^2 - 5x\end{aligned}$$

However, the exponential function has no zero since it never touch the x-axis. The only solutions obtained are from the second equation $x^2 - 5x = 0$

$$\begin{aligned}x^2 - 5x &= 0 \\x(x - 5) &= 0 \\x &= 0, 5\end{aligned}$$