Section 3.1 (Polynomial Functions and Models)

A polynomial is the finite sum of terms of the form ax^k such that $a \in \mathbb{R}$, and k is a whole number. So in general, a polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

One can see that

a. the function $f(x) = 2x - 5x + (2 + i)\, x^{12}$ is a polynomial

b. the function $g(x) = 3\sqrt{x} + 11x^{45}$ is not a polynomial

Definition 1 *(Zeros or roots) If $p(x)$ is a function, and $p(r) = 0$ then r is called the root of the function $p(x)$.*

Theorem 2 *Properties of graph of polynomials*

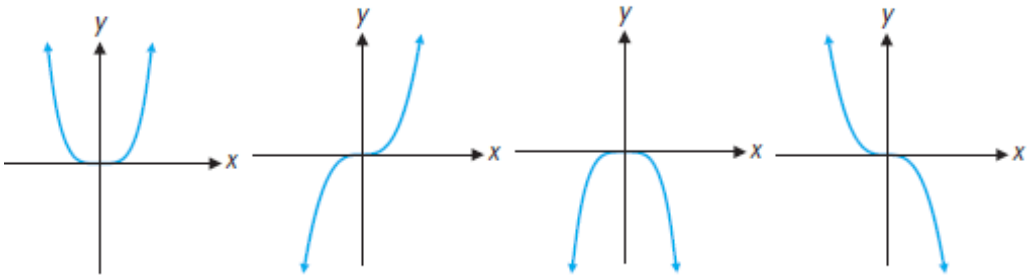
Let $P(x)$ be a polynomial of degree $n > 0$ with real coefficients. Then the graph of $P(x)$:

1. Is continuous for all real numbers
2. Has no sharp corners
3. Has at most n real zeros
4. Has at most $n - 1$ turning points
5. Increases or decreases without bound as x approaches ∞ and as x approaches $-\infty$

Theorem 3 *Left and right end behaviors of graphs of polynomials*

Let $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial function with real coefficients, $a_n \neq 0, n > 0$. If P has:

1. Positive leading coefficient, even degree: $P(x) \to \infty$ as $x \to \infty$ and $P(x) \to \infty$ as $x \to -\infty$ (like the graph of $y = x^2$).
2. Positive leading coefficient, odd degree: $P(x) \to \infty$ as $x \to \infty$ and $P(x) \to -\infty$ as $x \to -\infty$ (like the graph of $y = x^3$).
3. Negative leading coefficient, even degree: $P(x) \to -\infty$ as $x \to \infty$ and $P(x) \to -\infty$ as $x \to -\infty$ (like the graph of $y = -x^2$).
4. Negative leading coefficient, odd degree: $P(x) \to -\infty$ as $x \to \infty$ and $P(x) \to \infty$ as $x \to -\infty$ (like the graph of $y = -x^3$).

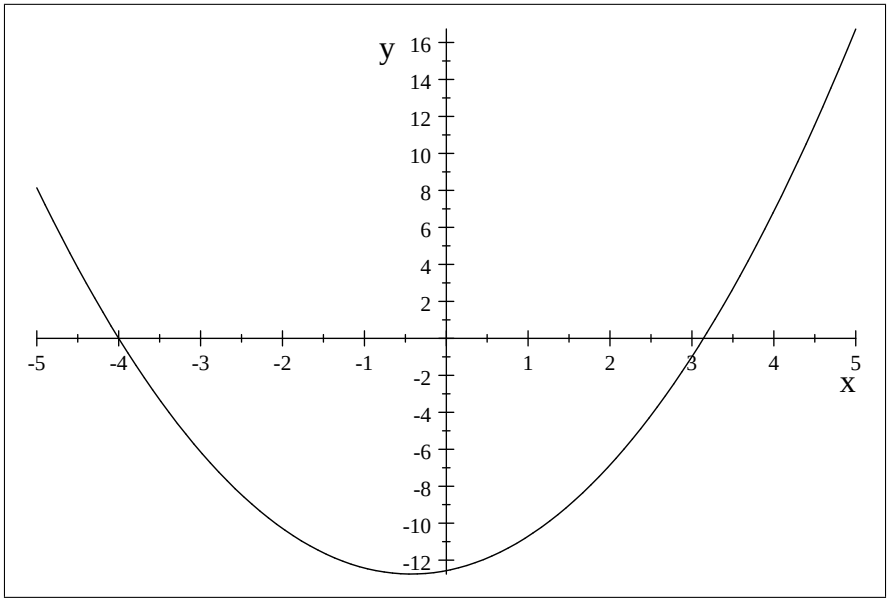


Case 1 Case 2 Case 3 Case 4

Example 4 *Find all the zeros of the polynomials* $P(x) = x\left(x^2 - 9\right)\left(x^2 + 4\right)$,
Solution is: $-3, 3, -2i, 2i, 0$

Example 5 *Find all the zeros of the polynomials* $P(x) = \left(x^2 - 5x + 6\right)\left(x^2 - 5x + 7\right)$,
Solution is: $\frac{1}{2}i\sqrt{3} + \frac{5}{2}, \frac{5}{2} - \frac{1}{2}i\sqrt{3}, 3, 2$

Example 6 *Sketch the graph of a polynomial which has for zeros* $x = -4, \pi$;
even degree, positive leading coefficient.



Example 7 *A rectangular storage container measuring 2 feet 2 feet by 3 feet is coated with protective coating of plastic of uniform thickness. Find the volume of plastic as a function of the thickness x of the coating*

If we consider the side of dimension 2×2, its volume is $4x$. If we consider the side which is 2×3, we obtain $6x$. Since we have 2 versions of the first one and 4 versions of the second, the volume obtained is

$$V = 2\,(4x) + 4\,(6x) = 32x$$