

Additional Equation-Solving Techniques

Example 1

1. Solve the equations

$$\begin{aligned}\sqrt{5x+6}+6 &= 0 \\ \sqrt{5x+6} &= -6\end{aligned}$$

We have no solution !

2. Solve algebraically $\sqrt{4x^2+8x+7}-x=1$, Solution is: $-1+i$

$$\begin{aligned}\sqrt{4x^2+8x+7}-x &= 1 \\ \sqrt{4x^2+8x+7} &= x+1 \\ \left(\sqrt{4x^2+8x+7}\right)^2 &= (x+1)^2 \\ 4x^2+8x+7 &= x^2+2x+1 \\ 4x^2-x^2+8x-2x+7-1 &= 0 \\ 3x^2+6x+6 &= 0 \\ x^2+2x+2 &= 0 \\ x &= \frac{-2 \pm \sqrt{4-8}}{2} \\ &= \frac{-2 \pm 2i}{2} \\ &= -1 \pm i\end{aligned}$$

3. Solve the equation $\sqrt{2x+3}=\sqrt{x^2-12}$

$$\begin{aligned}\sqrt{2x+3} &= \sqrt{x^2-12} \\ \left(\sqrt{2x+3}\right)^2 &= \left(\sqrt{x^2-12}\right)^2 \\ 2x+3 &= x^2-12 \\ x^2-2x-12-3 &= 0 \\ x^2-2x-15 &= 0 \\ x &= \frac{2 \pm \sqrt{4+4(15)}}{2} \\ x &= \frac{2 \pm 8}{2} \\ x &= 5, -3\end{aligned}$$

4. Solve the equation $y^{\frac{1}{2}}-3y^{\frac{1}{4}}+2=0$. By substitution, we let $x=y^{\frac{1}{2}}$, then

$$x^{\frac{1}{2}} = \left(y^{\frac{1}{2}}\right)^{\frac{1}{2}} = y^{\frac{1}{4}}$$

$$y^{\frac{1}{2}} - 3y^{\frac{1}{4}} + 2 = 0$$

$$x - 3x^{\frac{1}{2}} + 2 = 0$$

$$x - 3\sqrt{x} + 2 = 0$$

$$-3\sqrt{x} = -x - 2$$

$$3\sqrt{x} = x + 2$$

$$\sqrt{x} = \frac{x + 2}{3}$$

$$x = \left(\frac{x + 2}{3}\right)^2$$

$$= \frac{x^2 + 4x + 4}{9}$$

$$\frac{1}{9}x^2 + \frac{4}{9}x + \frac{4}{9} - x = 0$$

$$x^2 + 4x + 4 - 9x = 0$$

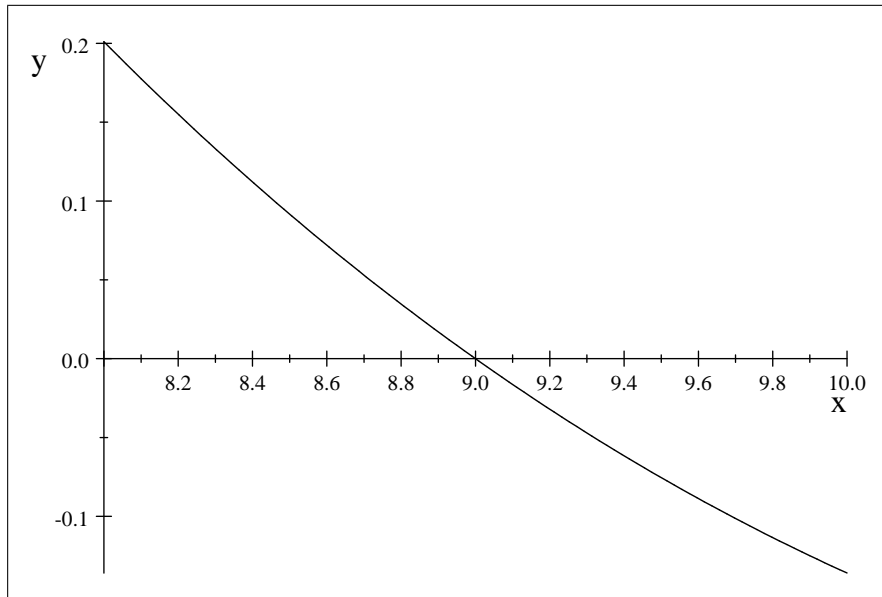
$$x^2 - 5x + 4 = 0$$

$$x = \frac{5 + \sqrt{25 - 16}}{2}, x = \frac{5 - \sqrt{25 - 16}}{2}$$

$$x = 4, x = 1$$

Thus, $x = y^{\frac{1}{2}}$ implies that $y = 16, 1$.

5. Solve by graphing the equation $m - 7\sqrt{m} + 12$



According to the graph, the solution is $m = 9$.