Section 2.5 Quadratic equations and models

Theorem 1 *(Zero factor theorem) Given the equation* $m \cdot n = 0$, *then we must either have* $m = 0$ *or* $n = 0$.

Theorem 2 *(The quadratic formula) Given an equation of the type* $ax^2 + bx + c = 0$, *we obtain the solutions as follows*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

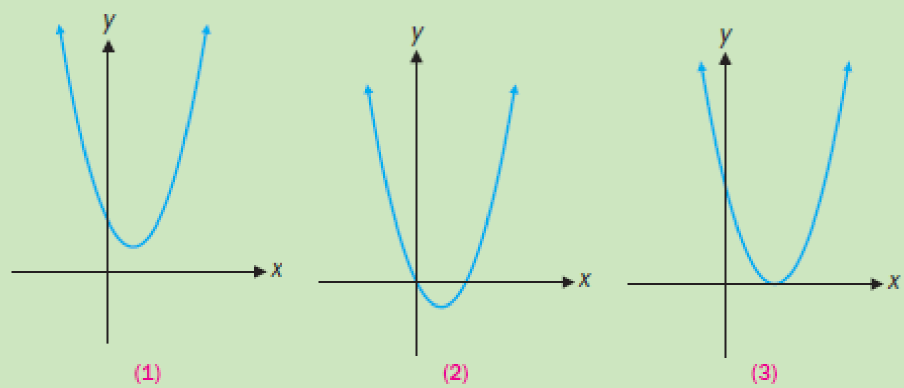
such that $\Delta = b^2 - 4ac$ *is called the discriminant.*

The discriminant of an equation give us valuable information about the type and number of solutions for the equation

(A) $D > 0$ **(B)** $D = 0$ **(C)** $D < 0$

In each of these three cases, what type of roots does the quadratic equation $f(x) = 0$ have?

Match each of the three cases with one of the following graphs.



Theorem 3 *Given* $ax^2 + bx + c = 0$, *we have*

$$\begin{cases} \text{2 complex solutions if } \Delta < 0 \\ \text{2 real solutions if } \Delta > 0 \\ \text{A unique solution if } \Delta = 0 \end{cases}$$

Example 4 *Solve* $(x - 8)(2x + 3) = 0$

$$\begin{aligned} x - 8 &= 0 \text{ or } 2x + 3 = 0 \\ x &= 8 \text{ or } x = -\frac{3}{2} \end{aligned}$$

We say the solution set is $\{8, -\frac{3}{2}\}$.

Example 5 *Solve by factoring* $11x = 2x^2 + 12$

Solution is: $4, \frac{3}{2}$

Example 6 *Solve by Completeting the square* $y^2 - 10y - 3$

$$\begin{aligned} y^2 - 10y - 3 &= y^2 - 10y + 25 - 3 - 25 \\ &= \left(y^2 - 10y + 25\right) - 3 - 25 \\ &= (y-5)^2 - 28 \end{aligned}$$

$$\begin{aligned} (y-5)^2 - 28 &= 0 \\ (y-5)^2 &= 28 \\ y - 5 &= \pm\sqrt{28} \\ y &= \pm\sqrt{28} + 5 \\ &= 2\sqrt{7} + 5 \end{aligned}$$

Example 7 *Solve by using the quadratic formula* $7x^2 + 6x + 4 = 0$*, Solution is:* $\frac{1}{7}i\sqrt{19} - \frac{3}{7}, -\frac{1}{7}i\sqrt{19} - \frac{3}{7}$

$$\begin{aligned} a &= 7, b = 6, c = 4 \\ x &= \frac{-6 \pm \sqrt{36 - 4 \times 28}}{14} \\ &= -\frac{3}{7} + \frac{1}{7}i\sqrt{19} \end{aligned}$$

Example 8 *Use the discriminant to determine the number and type of zeros (solutions)* $0.3x^2 + 3.6x + 10.8 = 0$

$$\begin{aligned} a &= 0.3, b = 3.6, c = 10.8 \\ \Delta &= (3.6)^2 - (4\,(10.8)\,0.3) \\ &= 0 \end{aligned}$$

Thus, we have a unique solution.