2.3 Quadratic Functions

Definition 1 *A **quadratic function** (written in general form) is a function of the type*

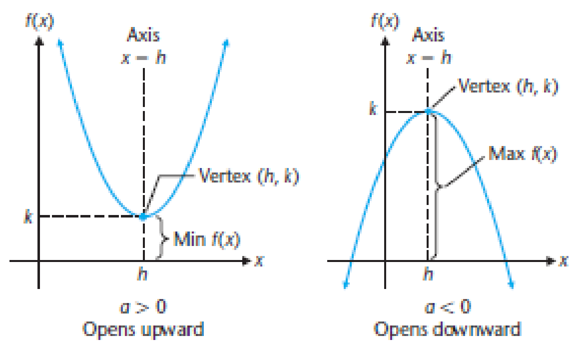
$$f(x) = ax^2 + bx + c$$

where a is a nonzero real value. A quadratic is also called a polynomial of degree 2. *The graph of a quadratic is called a "parabola".*

Definition 2 *Vertex form. A quadratic function could also be written as*

$$f(x) = a(x - h)^2 + k$$

such that the point (h, k) *is the vertex of the parabola, and the axis of symmetry is* $x = h$



Definition 3 *Square completion. To complete the square of the quadratic expression*

$$x^2 + bx$$

we need to add $\left(\frac{b}{2}\right)^2$ *to obtain a perfect square. In other other words,*

$$\begin{aligned} x^2 + bx + \left(\frac{b}{2}\right)^2 &= x^2 + 2\frac{b}{2}x + \left(\frac{b}{2}\right)^2 \\ &= \left(x + \frac{b}{2}\right)^2. \end{aligned}$$

Definition 4 *The x-coordinates of the vertex of the parabola* $f(x) = ax^2 + bx + c$ *is given by*

$$x = -\frac{b}{2a}.$$

Example 5 *Using graph transformations on* $g(x) = x^2$, *sketch the function* $f(x) = 2(x - 3)^2 + 4$

Example 6 *Complete the square*

$$x^2 - \frac{3}{4}x$$

Example 7 *Find the vertex form of the parabolas* $f(x) = x^2 - 8x + 4, g(x) = -3x^2 + 8x - 5$

$$\begin{aligned} f(x) &= x^2 - 8x + 16 + 4 - 16 \\ &= (x-4)^2 + 4 - 16 \\ &= (x-4)^2 - 12 \end{aligned}$$

$$\begin{aligned} g(x) &= -3x^2 + 8x - 5 \\ &= -3\left(x^2 - \frac{8}{3}x\right) - 5 \\ &= -3\left(x^2 - \frac{8}{3}x + \left(\frac{8}{2*3}\right)^2\right) - 5 + 3\left(\frac{8}{2*3}\right)^2 \\ &= -3\left(x^2 - \frac{8}{3}x + \frac{16}{9}\right) - 5 + \frac{16}{3} \\ &= -3\left(x - \frac{4}{3}\right)^2 + \frac{1}{3}. \end{aligned}$$

Example 8 *The height of a projectile traveling through space is given a function of time t by*

$$h(t) = 200 - 16t^2$$

When will the projectile hit the ground.

$$\begin{aligned} h(t) &= 0 \textit{ implies } 200 - 16t^2 = 0 \\ -16t^2 &= -200 \\ 16t^2 &= 200 \\ t^2 &= \frac{200}{16} = \frac{25}{2} \\ t &= \pm\sqrt{\frac{25}{2}} \end{aligned}$$

Since time may not be negative, then $t = \sqrt{\frac{25}{2}} = 3.5355$ *seconds.*