Section 2.3 Quadratic Equations

DEFINITION 1 Quadratic Functions

If a, b, and c are real numbers with $a \neq 0$, then the function

$$f(x) = ax^2 + bx + c$$

is called a **quadratic function** and its graph is called a **parabola.*** This is known as the **general form** of a quadratic function.

Definition

The Vertex Form of a Quadratic Function

We will begin our detailed study of quadratic functions by examining some in a special form, which we will call the **vertex form:***

$$f(x) = a(x - h)^2 + k$$

Definitions

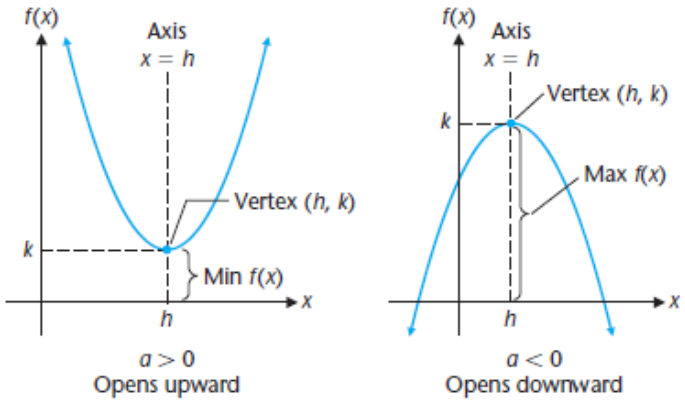
PROPERTIES OF A QUADRATIC FUNCTION AND ITS GRAPH

Given a quadratic function in vertex form

$$f(x) = a(x - h)^2 + k \qquad a \neq 0$$

we summarize general properties as follows:

1. The graph of f is a parabola:



$a > 0$
Opens upward

$a < 0$
Opens downward

2. Vertex: (h, k) (parabola rises on one side of the vertex and falls on the other).
3. Axis (of symmetry): $x = h$ (parallel to y axis).
4. $f(h) = k$ is the minimum if $a > 0$ and the maximum if $a < 0$.
5. Domain: all real numbers; range: $(-\infty, k]$ if $a < 0$ or $[k, \infty)$ if $a > 0$.
6. The graph of f is the graph of $g(x) = ax^2$ translated horizontally h units and vertically k units.

COMPLETING THE SQUARE

To **complete the square** of the quadratic expression

$$x^2 + bx$$ Leading coefficient 1

add the square of one-half the coefficient of x; that is, add

$$\left(\frac{b}{2}\right)^2 \quad \text{or} \quad \frac{b^2}{4}$$

The resulting expression can be factored as a perfect square:

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Examples

Complete the square for each of the following:

(A) $x^2 - 8x$ (B) $x^2 + \frac{7}{4}x$ (C) $x^2 - \frac{2}{3}x$

Example

Find the vertex form of $g(x) = x^2 + 10x - 1$, then write the vertex

FINDING THE VERTEX OF A PARABOLA

When a quadratic function is written in the form $f(x) = ax^2 + bx + c$, the first coordinate of the vertex can be found using the formula

$$x = -\frac{b}{2a}$$

The second coordinate can then be found by evaluating f at the first coordinate.

Example

Find the vertex of the parabola $g(x) = 2x^2 - 16x + 10$.