

**Math 141-01**  
**Section 1.4**  
**Functions: Graphs and Transformations**  
Lecture notes by Vignon S. Oussa

**Library functions:**

- $f(x) = x$
- $f(x) = |x|$
- $f(x) = x^2$
- $f(x) = x^3$
- $f(x) = \sqrt{x}$
- $f(x) = \sqrt[3]{x}$

**Vertical shifting**

Given a function  $f(x)$ , the graph of  $g(x) = f(x) + a$ , is obtained by shifting vertically the graph of  $f$ ,  $a$  units. If  $a$  is positive, the shift is upward and if  $a$  is negative, the shift is downward.

**Horizontal shifts**

Given a positive number,  $a$ , and a function  $f(x)$ , the graph of the function  $g(x) = f(x - a)$  is obtained by shifting horizontally the graph of  $f$   $a$  units to the right, and the graph of the function  $g(x) = f(x + a)$  is obtained by shifting horizontally the graph of  $f$   $a$  units to the left.

**Reflection**

Given a function  $f(x)$ , the graph of  $-f(x)$  is obtained by reflecting the graph of  $f$  with respect to the x-axis. The graph of  $f(-x)$  is obtained by reflecting the graph of  $f$  with respect to the y-axis.

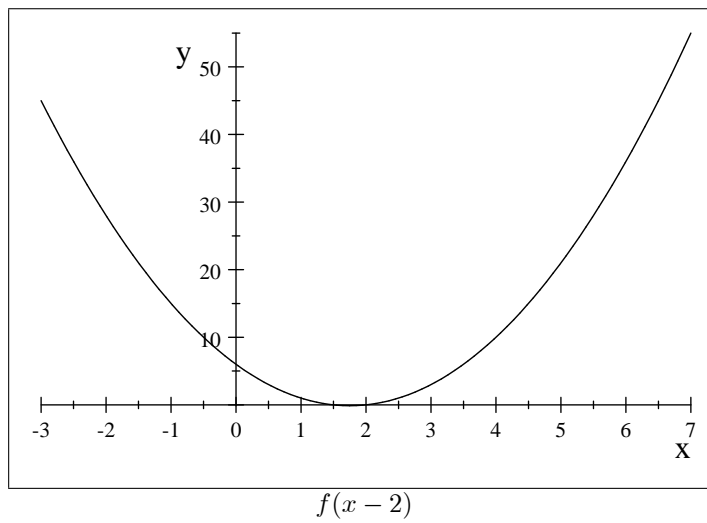
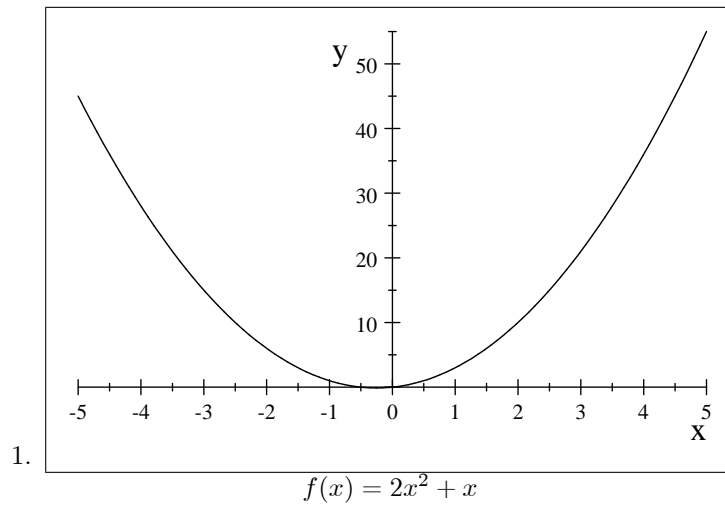
**Even and odd**

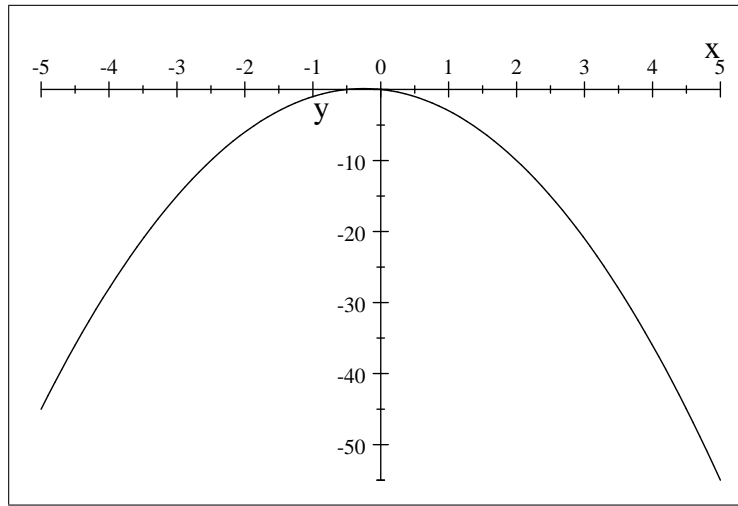
A function  $f$  is even if and only if  $f(-x) = f(x)$ .  $f$  is odd if and only if  $f(-x) = -f(x)$ . The graph of an even function is symmetrical with respect to the y-axis, and the graph of an odd function is symmetrical with respect to the x-axis.

**Examples:**

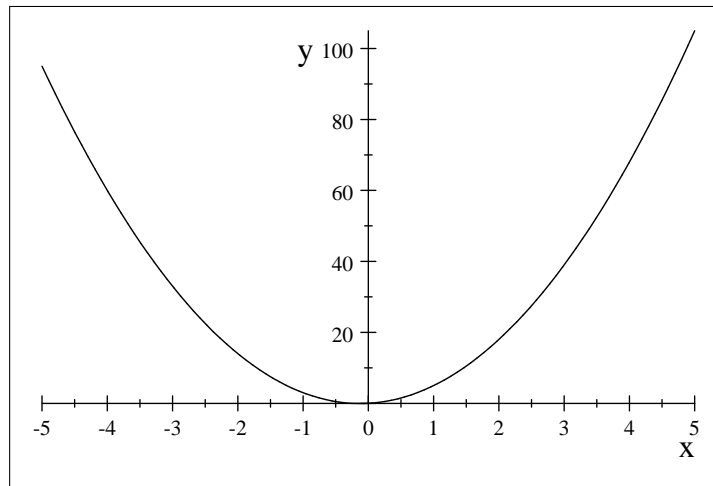
1. Given  $f(x) = 2x^2 + x$ , plot the graph of the following functions  $f(x) - 2$ ,  $f(x) + 2$ ,  $f(x - 2)$ ,  $f(x + 2)$ ,  $-f(x)$ ,  $f(-x)$ ,  $2f(x)$ , and  $f(2x)$
2. By using graph transformations, plot the following function using a library function:  $f(x) = -\sqrt{x - 2} + 2$
3. Determine if the functions defined by  $f(x) = x^3 + x^2$ ,  $x^5$ ,  $x^8 - x^4$  are either even, odd or neither.

**Partial solutions:**



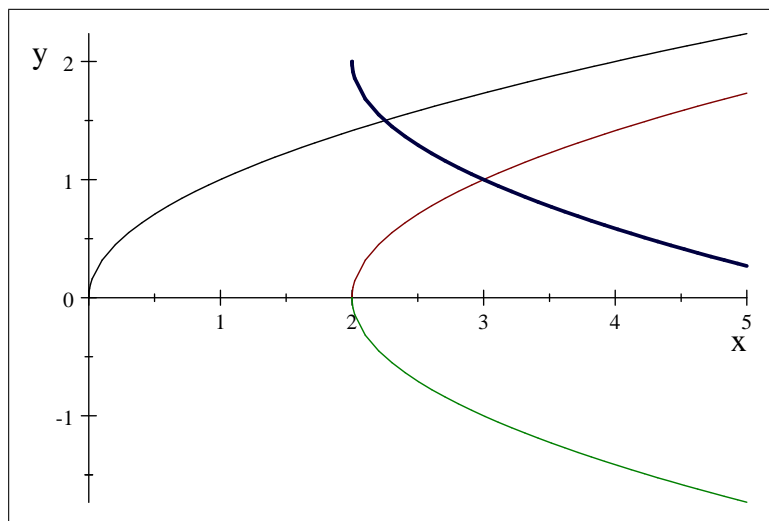


$f(-x)$



$f(2x)$

2. By shifting the graph of  $\sqrt{x}$  two unit to the right, then we reflect with respect to the y-axis and shift vertically 3 units  $-\sqrt{x-2} + 2, \sqrt{x}$



3. Determining if the functions defined by  $f(x) = x^3 + x^2, x^5, x^8 - x^4$  are either even, odd or neither.  $f(x) = x^3 + x^2$  is neither because

$$\begin{aligned} f(-x) &= (-x)^3 + (-x)^2 \\ &= -x^3 + x^2 \neq f(x) \\ &\neq -f(x) = -x^3 - x^2 \end{aligned}$$

$f(x) = x^5$  is odd because,

$$\begin{aligned} f(-x) &= (-x)^5 \\ &= -x^5 \\ &= -f(x). \end{aligned}$$

$f(x) = x^8 - x^4$  is even since,

$$\begin{aligned} f(-x) &= (-x)^8 - (-x^4) \\ &= x^8 - x^4 \\ &= f(x) \end{aligned}$$

**Homework:** Page 79#5, 7, 9, 11, 15, 21, 25, 27, 29, 41, 45, 47, 52