Section 1.2
Functions

By definition, a **function** is a relation in which every element in the domain corresponds to one and only one element in the range. Alternatively, a function is a set of ordered pairs with the property that no two pairs have the same element in the first component and different element in the second component. The set of all elements in the first component is called the **domain** of the function, and set of all element in the second component is called the **range** of the function.
The **vertical line test**, is a test applied to the graph of a function which tell us whether the graph represents a function or not.If there exits any vertical line that meets the graph of the function at more than one point, the relation is not a function. Otherwise, it is a function.

Examples

1. Determine if the following relation is a function

$$y = 4 + \sqrt{x+4}$$

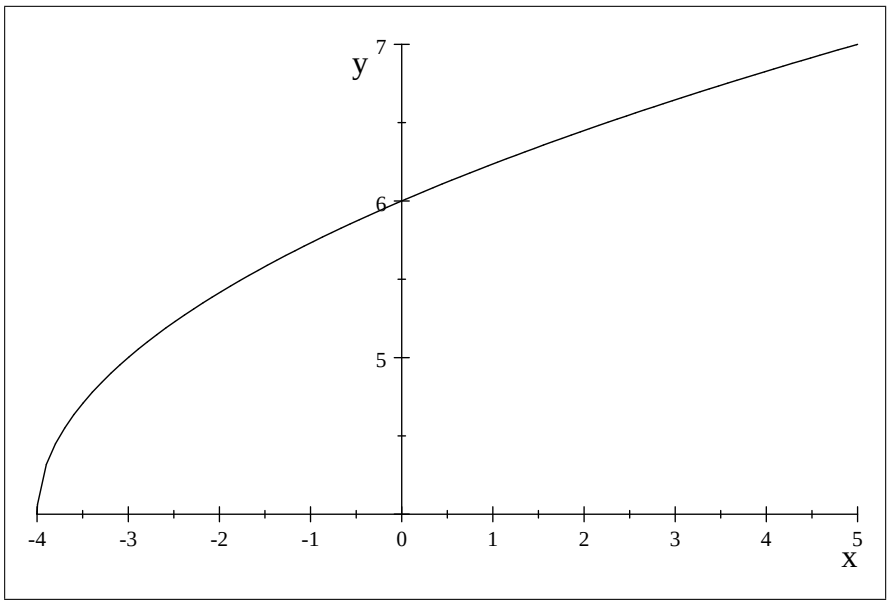
2. Find the domain of the functions: $f(x) = 4+\sqrt{x+4}, g(x) = \frac{15}{v^2-16}, h(x) = \frac{\sqrt{x-3}}{x+2}$

3. Given the function $f(x) = \frac{15}{x-2}, g(x) = 2x^2+3x-6, h(x) = \frac{1}{\sqrt{x-1}}$, compute $f(2), g(0), h(3)$

4. Given the function $f(x) = x^2 + 2x - 1$, compute $f(2), f(2+h)$, and $\frac{f(2+h)-f(2)}{h}$

Partial Solutions

1. Plotting the graph of the relation $y = 4 + \sqrt{x+4}$,we obtain



Clearly, the no vertical line meets the graph of this function at more than one point. Thus, by the vertical line test, the relation is a function.

2. Finding the domain of $f(x) = 4 + \sqrt{x+4}, g(x) = \frac{15}{v^2-16}$ Let D_f denotes the domain of $f(x)$

$$\begin{aligned} D_f &= \{x \in \mathbb{R} : x + 4 \geq 0\} \\ x + 4 &\geq 0 \Longrightarrow x \geq -4 \\ &\Rightarrow x \in [-4, \infty) \end{aligned}$$

Let D_g denotes the domain of the function $g(x)$

$$\begin{aligned} D_g &= \{v \in \mathbb{R} : v^2 - 16 \neq 0\} \\ v^2 - 16 &= 0 \Rightarrow (v-4)(v+4) = 0 \\ &\Rightarrow v - 4 = 0 \text{ or } v + 4 = 0 \\ &\Rightarrow v = 4 \text{ or } v = -4 \end{aligned}$$

Thus,

$$\begin{aligned} D_g &= \{v \in \mathbb{R} : v^2 - 16 \neq 0\} \\ &= \{v \in \mathbb{R} : v^2 - 16 \neq 0\} \\ &= \{v \in \mathbb{R} : v \neq 4 \text{ and } v \neq -4\}. \\ &= (-\infty, -4) \cup (-4, 4) \cup (4, \infty) \end{aligned}$$

3. Given the function $f(x) = \frac{15}{x-2}, g(x) = 2x^2 + 3x - 6, h(x) = \frac{1}{\sqrt{x-1}}$, we

compute $f(2), g(0), h(3)$

$$\begin{aligned} f(2) &= \frac{15}{2-2} = \frac{15}{0} \text{ is undefined} \\ g(0) &= 2 \cdot 0^2 + 3 \cdot 0 - 6 = -6 \\ h(3) &= \frac{1}{\sqrt{3-1}} = \frac{1}{\sqrt{2}} \end{aligned}$$

4. Given the function $f(x) = x^2+2x-1$, we compute $f(2+h)$, and $\frac{f(2+h)-f(2)}{h}$.

$$\begin{aligned} f(2+h) &= (2+h)^2 + 2(2+h) - 1 \\ &= 4 + 4h + h^2 + 4 + 2h - 1 \\ &= h^2 + 6h + 7. \end{aligned}$$

$$\begin{aligned} \frac{f(2+h) - f(2)}{h} &= \frac{h^2 + 6h + 7 - \left(2^2 + 2 \cdot 2 - 1\right)}{h} \\ &= \frac{h^2 + 6h}{h} \\ &= \frac{(h+6)\,h}{h} \\ &= (h+6)\,. \end{aligned}$$

Homework:

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