Fall 2011
Precalculus
Project F

Consider the function

$$F(x) = \cos(x)^6 - \sin(x)^6.$$

We want to linearize $F(x)$. In other words, we want to write the function F without powers.

1. Establish the identity

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$
$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

2. Using part 1, linearize $\cos^4(x)$ and $\sin^4(x)$.

3. Using part 2, linearize $\cos^6(x)$ and $\sin^6(x)$

4. Finally, show that

$$F(x) = \frac{15}{16}\cos(2x) - \frac{1}{16}\cos(6x)$$

5. Plot both

$$y = \cos(x)^6 - \sin(x)^6$$

and

$$y = \frac{15}{16}\cos(2x) - \frac{1}{16}\cos(6x)$$

together. What do you observe?