Section 1.5 (graph Techniques ; Transformations)

Given $f(x)$

$f(x+h)$ $h > 0$	horizontal shift to the left by h units
$f(x-h)$ $h > 0$	horizontal shift to the right by h units
$f(x)+k$ $k > 0$	vertical shift upward by k units
$f(x)-k$ $k > 0$	vertical shift downward by k units

Compressions and Stretches

When the right side of a function $y = f(x)$ is multiplied by a positive number a, the graph of the new function $y = a f(x)$ is obtained by multiplying each y-coordinate on the graph of $y = f(x)$ by a.

- If $0 < a < 1$, the new graph is vertically <u>compressed</u>
- If $a > 1$, the new graph is vertically <u>stretched</u>

Recall

Given $y = f(x)$, x is called the argument of $y = f(x)$

If the argument x of a function $y = f(x)$ is multiplied by a positive number a, the graph of the new function $y = f(ax)$ is obtained by multiplying each x-coord of $y = f(x)$ by $\frac{1}{a}$

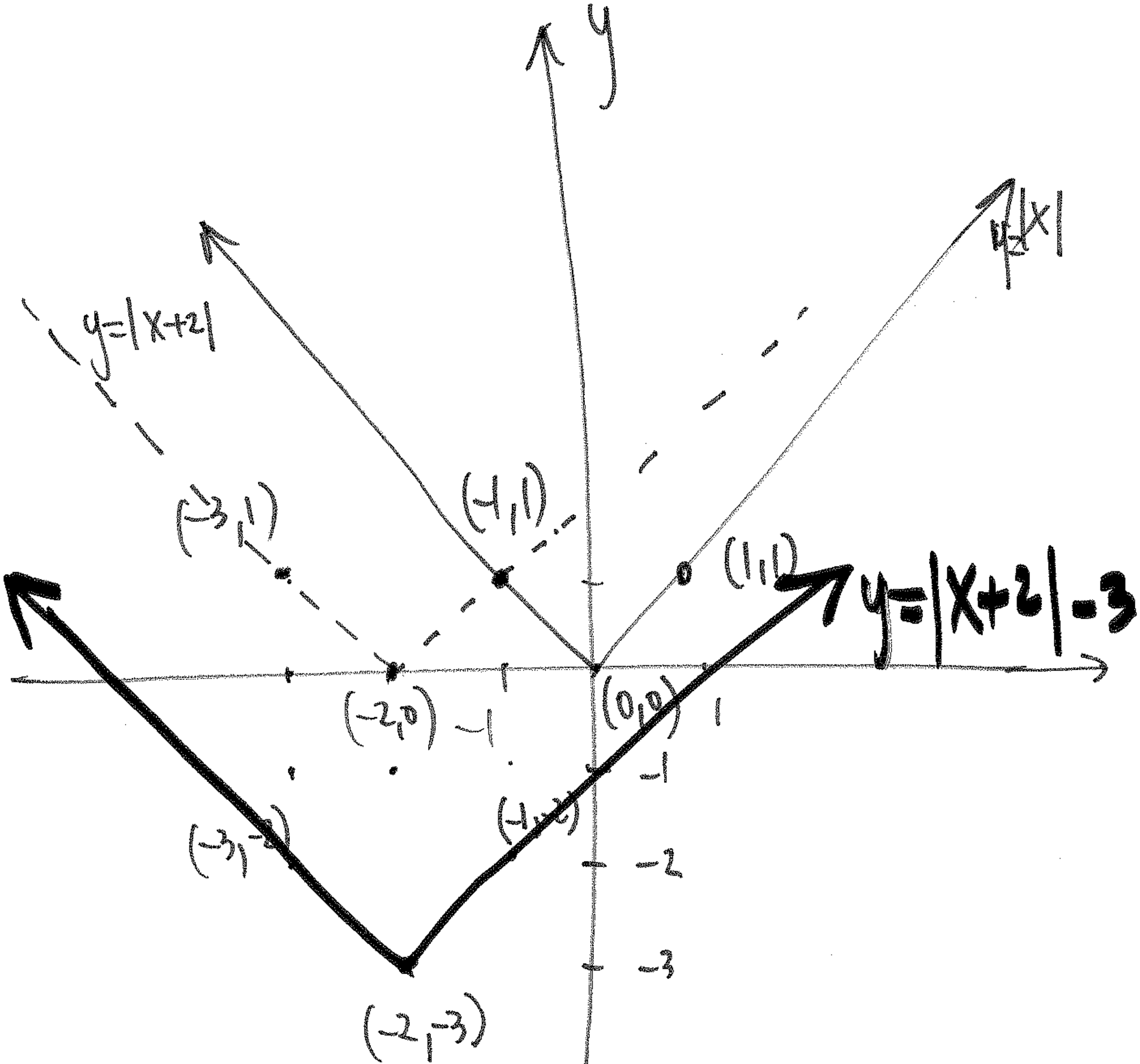
- If $a > 1$, we will apply a horizontal compression
- If $0 < a < 1$, we will apply a horizontal stretch.

Reflection about the x-axis

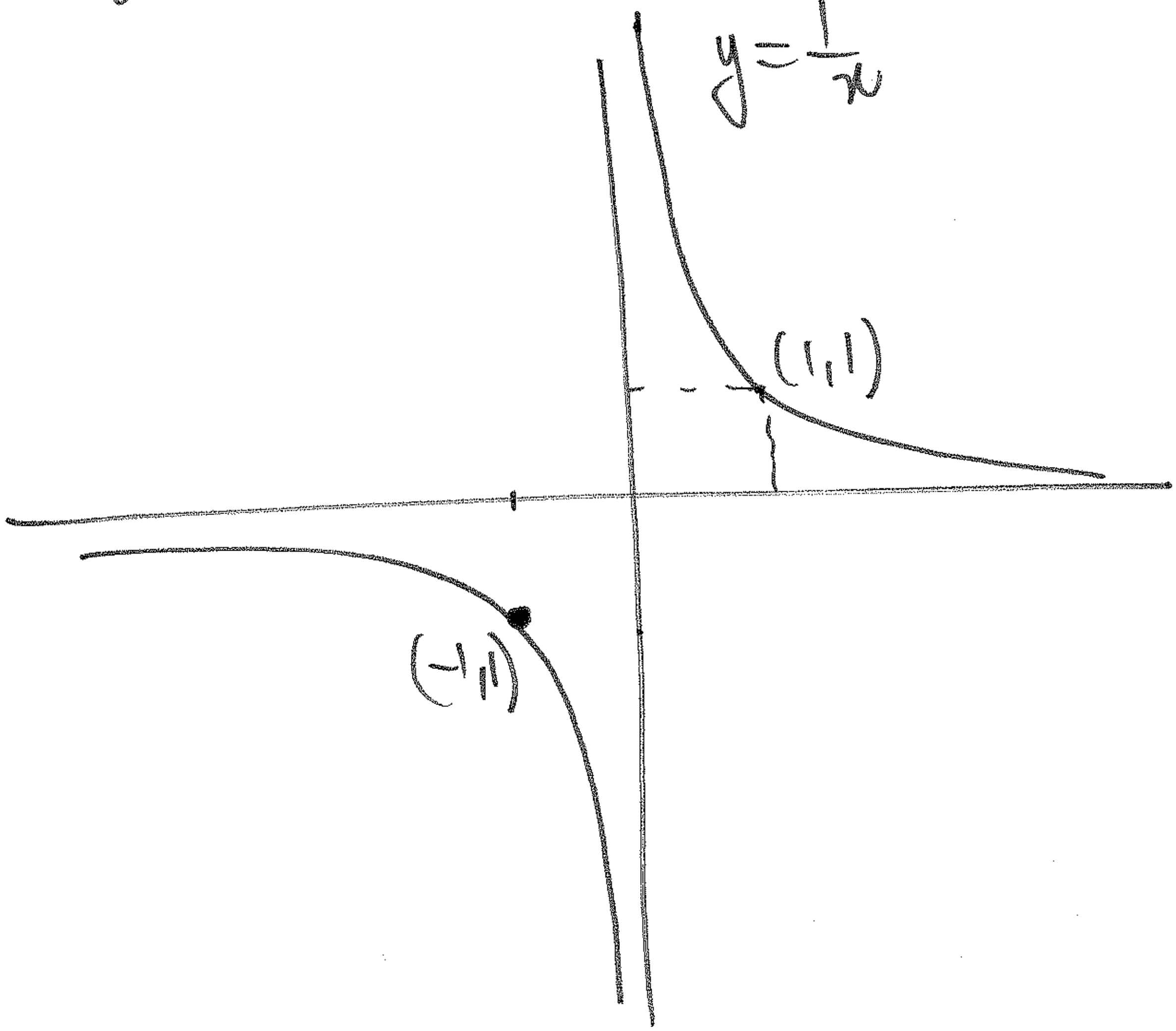
When the right side of the function $y = f(x)$ is multiplied by -1, the graph of the new function $y = -f(x)$ is the reflection about the x-axis of the graph of the new function $y = f(x)$

When the graph of the function $y = f(x)$ is known, the graph of the new function $y = f(-x)$ is the reflection about the y-axis of the graph of the function $y = f(x)$.

Sketch $y = |x+2| - 3$



$$y = \frac{3}{x-2} + 1$$



2.3 Quadratic functions and their zeros

A quadratic function

$$f(x) = ax^2 + bx + c$$

a, b, c are real numbers, $a \neq 0$.

Ex $3x^2 + 5x + 1$ is a quadratic

$a = 3$, $b = 5$, $c = 1$

A quadratic equation is an equation of the type

$$3x^2 + 5x + 1 = 0$$

Rule 1 $\quad x^2 = p \quad , p > 0$

$$x = \sqrt{p} \quad \text{or} \quad x = -\sqrt{p}\ .$$

This is called the square root method

Ex $\quad$ Find the zeros of $f(x) = x^2 - 12$

$$x^2 - 12 = 0$$

$$x^2 = 12$$

$\sqrt{12} = \sqrt{4 \times 3}$

by the square root method

$$x = \sqrt{12} \quad \text{or} \quad x = -\sqrt{12}$$

$$= 2\sqrt{3} \quad \text{or} \quad x = -2\sqrt{3}$$

Ex 1 Find the zeros by factoring

$$f(x) = 3x^2 + 4x - 4$$

$$3x^2 + 4x - 4 = 0$$

$$3x^2 + 6x - 2x - 4 = 0$$

$$3x(x+2) - 2(x+2) = 0$$

$$(x+2)(3x-2) = 0$$

$$x + 2 = 0 \quad \text{or} \quad 3x - 2 = 0$$

$$x = -2 \quad \text{or} \quad x = \frac{2}{3}$$

Ex2 Find the zeros by using the square root method

$$f(x) = (4x-1)^2 - 16$$

$$(4x-1)^2 - 16 = 0$$

$$(4x-1)^2 = 16$$

$$4x-1 = \sqrt{16} \quad \text{or} \quad 4x-1 = -\sqrt{16}$$

$$4x-1 = 4 \quad \text{or} \quad 4x-1 = -4$$

$$\frac{4x}{4} = \frac{5}{4} \quad \text{or} \quad \frac{4x}{4} = \frac{-3}{4}$$

$$x = \frac{5}{4} \quad \text{or} \quad x = -\frac{3}{4}$$

Ex 3 Find the zeros by completing the square $f(x) = x^2 + 4x - 10$

$$x^2 + 4x - 10 = 0$$

$$(x^2 + 4x + 4) - 4 - 10 = 0$$

$$(x+2)^2 - 14 = 0$$

$$(x+2)^2 = 14$$

$x + 2 = \sqrt{14}$ or $x + 2 = -\sqrt{14}$ by the square root Method

$$\boxed{x = \sqrt{14} - 2 \text{, or } x = -\sqrt{14} - 2}$$

Solve by completing the square

$$x^2 + \frac{11}{2}x + 3 = 0$$

$$x^2 + \frac{11}{2}x + \frac{121}{16} - \frac{121}{16} + 3 = 0$$

$$\left(x + \frac{11}{4}\right)^2 - \frac{121}{16} + 3 = 0$$

$$\left(x + \frac{11}{4}\right)^2 - \frac{121}{16} + \frac{48}{16} = 0$$

$$\left(x + \frac{11}{4}\right)^2 - \frac{73}{16} = 0$$

$$\left(x + \frac{11}{4}\right)^2 = \frac{73}{16}$$ square root method

$$x + \frac{11}{4} = \frac{\sqrt{73}}{4} \quad \text{or} \quad x + \frac{11}{4} = -\frac{\sqrt{73}}{4}$$

$$\boxed{x = \frac{\sqrt{73}}{4} - \frac{11}{4} \quad \text{or} \quad x = -\frac{\sqrt{73}}{4} - \frac{11}{4}}$$

Find the zeros of $f(x) = 3x^2 - 5x - 7$
by using the quadratic formula

$a = 3, \ b = -5, \ c = -7$

$$x = \frac{5 \pm \sqrt{25 - 4(3)(-7)}}{6}$$

$$\begin{array}{r} 21 \\ \underline{4} \\ 84 \end{array}$$

$$= \frac{5 \pm \sqrt{25 + 84}}{6}$$

$$\begin{array}{r} 84 \\ \underline{25} \\ 109 \end{array}$$

$$= \boxed{\frac{5 \pm \sqrt{109}}{6}}$$