Math 141- Section 01 Fall 2010 - Final Exam
Name

Show all your work in a precise and clear manner to receive credits.

1. **(10 points)** Compute $\sin\left(\arctan\left(-\frac{4}{5}\right)\right)$

2. **(10 points)** Solve the equation

$$2\sin^2\left(\frac{x}{2}\right) - 3\sin\left(\frac{x}{2}\right) + 1 = 0$$

3. **(10 points)** Convert $1 - i\sqrt{3}$ to polar form

4. **(10 points)** Find $\left(2e^{i15^\circ}\right)^4$ using De Moivre's theorem and write the final answer in exact rectangular form

5. **(10 points)** Find all the $6th$ roots of -1

Multiple choice part: 5 points each, (without showing your work, choose the correct answers)

1. The period of $\cos\frac{x}{5}$ is

   (a) 5π

   (b) -5π

   (c) 10π

   (d) -10π

   (e) 20π

   (f) $\frac{\pi}{5}$

   (g) $-\frac{\pi}{5}$

2. In which quadrant is $\tan x$ negative

   (a) I, II

   (b) II, IV

   (c) I, II, III

   (d) II, IV, III

   (e) IV

   (f) I, IV

   (g) III, II

3. If 2 angles are complementary then

   (a) They must have the same cosine

   (b) They must have the same sine

   (c) They must have opposite cosine values

   (d) The cosine of one must be equal to the sine of the other

   (e) The sine of one must be equal to the cosine of the other

   (f) Both d and e are true

   (g) None of the above

4. There exists a function which is both even and odd

   (a) True

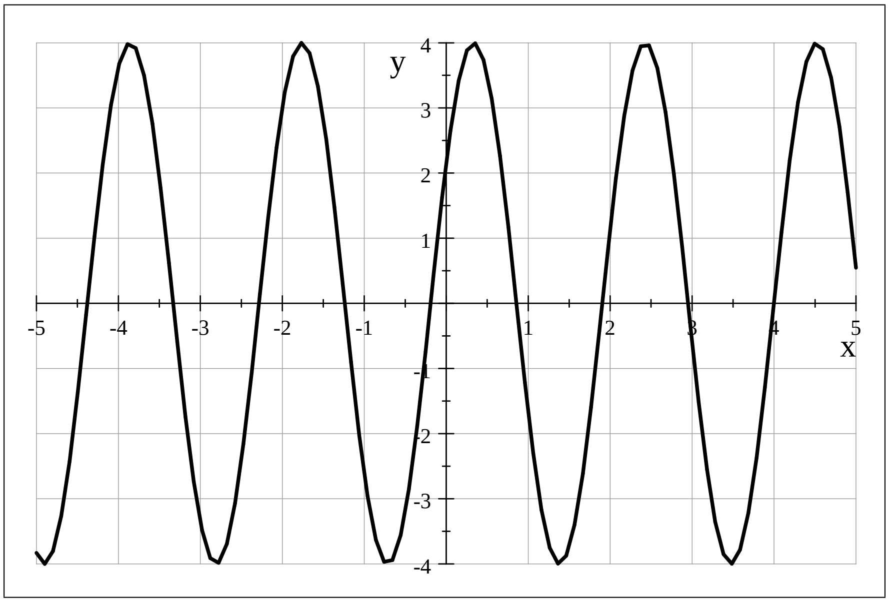
   (b) False

   (c) Both true and false

5. When defining $\arcsin x$ we must take the restriction of $\sin x$ on

(a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$
(c) $[0, \pi]$
(d) $[-\pi, \pi]$
(e) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \cup [-\pi, \pi]$
(f) $\left[\frac{\pi}{2}, -\frac{\pi}{2}\right]$
(g) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

6. Given the graph of the following function $f(x)$ below



(a) The amplitude is -4 and the period $-\frac{\pi}{3}$
(b) The amplitude is -4 and the period $\frac{2\pi}{3}$
(c) The amplitude is 4 and the period $-\frac{2\pi}{3}$
(d) The amplitude is 4 and the period $\frac{2\pi}{3}$
(e) The amplitude is -4 and the period $-\frac{2\pi}{3}$
(f) The amplitude is 2 and the period $\frac{2\pi}{3}$
(g) The amplitude is -2 and the period $\frac{2\pi}{3}$

7. $\cos^2 x + \sin^2(x)$ is equal to

(a) -1
(b) 2
(c) 1
(d) -2
(e) i

(f) $\sin(x$

(g) $\cos x$

8. $\cos\left(x - \frac{3\pi}{2}\right)$ is equal to

   (a) $\sin x$
   (b) $\cos x$
   (c) $-\cos x$
   (d) $\cos 2x$
   (e) $-\sin x$
   (f) $\tan x$
   (g) $\cot x$

9. $\cos(x - y)$ is equal to

   (a) $\cos x \cos y - \cos x \cos y$
   (b) $\cos x \cos y - \sin x \sin y$
   (c) $\sin x \sin y - \cos x \cos y$
   (d) $\cos x \cos y + \sin x \sin y$
   (e) $\cos x \sin y + \cos y \sin y$
   (f) $\sin x \cos y - \sin y \cos x$
   (g) None of the above

10. $\tan(x - y)\tan(x + y)$ is equal to

   (a) $\frac{\tan^2 x - \tan^2 y}{\tan^2 x \tan^2 y - 1}$
   (b) $-\frac{\tan^2 x - \tan^2 y}{\tan^2 x \tan^2 y - 1}$
   (c) $-\frac{\tan^2 x - \tan^2 y}{\tan^2 x \tan^2 y + 1}$
   (d) $-\frac{\tan^2 x + \tan^2 y}{\tan^2 x \tan^2 y - 1}$
   (e) $-\frac{\tan^2 x - \tan^2 y}{\tan^2 x \tan^2 y - 2}$
   (f) $\frac{\tan^2 x - \tan^2 y}{\tan^2 y \tan^2 y - 1}$
   (g) $-\frac{-\tan^2 x - \tan^2 y}{\tan^2 x \tan^2 y - 1}$

11. $\tan\left(\frac{\pi}{2} - x\right)\sin\left(\frac{\pi}{2} - x\right)\cos\left(\frac{\pi}{2} - x\right)$ is equal to

   (a) $\cos^2 x$
   (b) $1 - \sin^2 x$
   (c) $\cot x \cos x \sin x$

(d) a and b

(e) a and c

(f) b and c

(g) a, b and c

12. In class the instructor stated that multiplying a complex number belonging to the unit circle is equivalent with

(a) Rotating clockwise the complex number by the value of its argument

(b) Rotating counterclockwise the complex number by the value of its argument

(c) Rotating clockwise the complex number by the value of its modulus

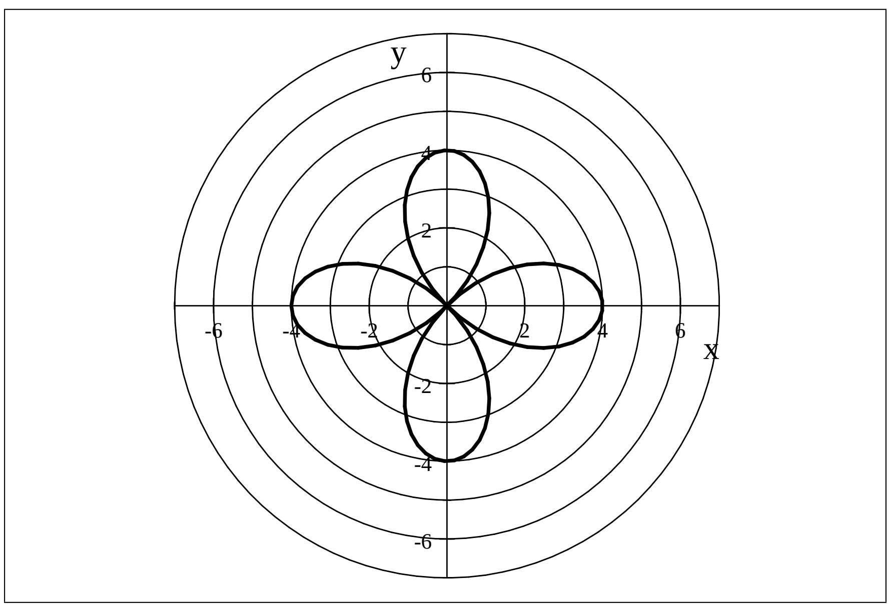
(d) Rotating clockwise the complex number by the double value of its argument

(e) Rotating counterclockwise the complex number by the double value of its argument

(f) Dilating the complex number by the double value of its modulus

(g) Dilating the complex number by the value of its modulus

13. Which function has for graph in polar coordinates the following:



(a) $4\cos 2\theta$

(b) $4\sin 2\theta$

(c) $4\cos 4\theta$

(d) $4\sin 4\theta$

(e) $3\sin 2\theta$

(f) $8\cos 8\theta$

(g) $8\sin 8\theta$

14. The 3th roots $-i$ are

(a) $\frac{1}{2}i\sqrt{-3}+\frac{1}{2}, \frac{1}{2}-\frac{1}{2}i\sqrt{3}, -1$
(b) $\frac{1}{2}\sqrt{3}+\frac{1}{2}, \frac{1}{2}-\frac{1}{2}\sqrt{3}, -1$
(c) $\frac{1}{2}i\sqrt{3}+\frac{1}{2}, \frac{1}{2}-\frac{1}{2}\sqrt{3}, -1$
(d) $\frac{1}{2}\sqrt{3}+\frac{1}{2}, \frac{1}{2}-\frac{1}{2}i\sqrt{3}, -1$
(e) $\frac{1}{2}i\sqrt{3}+\frac{1}{2}, \frac{1}{2}-\frac{1}{2}i\sqrt{3}, 1$
(f) $\frac{1}{2}i\sqrt{3}+\frac{1}{2}, \frac{1}{2}-\frac{1}{2}i\sqrt{3}, -1$
(g) $\frac{1}{2}i\sqrt{3}+\frac{1}{2}, \frac{1}{2}-\frac{1}{2}e\sqrt{3}, -\frac{1}{2}$

15. According to DeMoivre's theorem $(r(\cos\theta + i\sin\theta))^n$ is equal to

(a) b and d
(b) $r^n(\sin n\theta + i\cos n\theta)$
(c) $r^n(\cos n\theta - i\sin n\theta)$
(d) $r^n(\cos n\theta + i\sin n\theta)$
(e) $r^n(\cos\theta + i\sin\theta)$
(f) $r^n(\cos\theta + i\sin n\theta)$
(g) b, d and f

16. Convert $r = -4\cos\theta$ to rectangular form

(a) $x^2 + y^2 = -4x$
(b) $x^2 + y^2 = 4x$
(c) $x^2 - y^2 = -4x$
(d) $x^2 + y^2 = -2x$
(e) $-x^2 + y^2 = -4x$
(f) $x^2 + y^2 = 16x$
(g) $x^2 + y^2 = -16x$

17. The fourth roots of $z = 1 + i$ are

$$\sqrt[8]{2}\sin\left(\frac{\pi}{16}\right) - i\sqrt[8]{2}\cos\left(\frac{\pi}{16}\right),$$
$$-\sqrt[8]{2}\cos\left(\frac{\pi}{16}\right) - i\sqrt[8]{2}\sin\left(\frac{\pi}{16}\right),$$
$$i\sqrt[8]{2}\cos\left(\frac{\pi}{16}\right) - \sqrt[8]{2}\sin\left(\frac{\pi}{16}\right),$$
$$\sqrt[8]{2}\cos\left(\frac{\pi}{16}\right) + i\sqrt[8]{2}\sin\left(\frac{\pi}{16}\right).$$

(a) True

(b) False

18. If the modulus of the complex number $z = x + iy$ is equal to 1 and its argument is equal to $\frac{47\pi}{4}$ then

(a) $z = \sqrt{2}\left(\frac{1}{2} - \frac{1}{2}i\right)$

(b) $z = 2\left(\frac{1}{2} - \frac{1}{2}i\right)$

(c) $z = -\sqrt{2}\left(\frac{1}{2} - \frac{1}{2}i\right)$

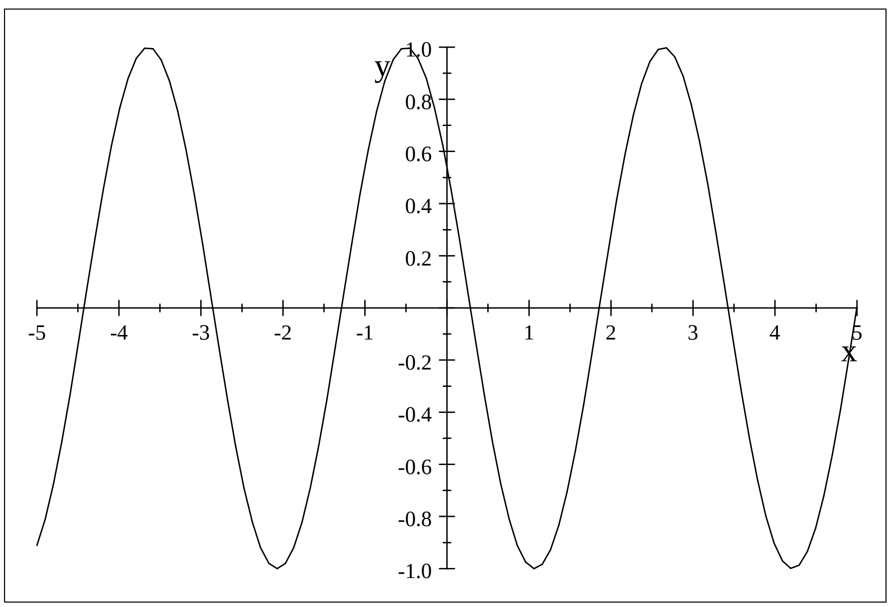
(d) $z = -2\left(\frac{1}{2} - \frac{1}{2}i\right)$

(e) $z = \sqrt{2}\left(\frac{1}{2} + \frac{1}{2}i\right)$

(f) $z = \sqrt{2}\left(-\frac{1}{2} - \frac{1}{2}i\right)$

(g) $z = \sqrt{2i}\left(\frac{1}{2} - \frac{1}{2}i\right)$

19. The following



is the graph of

(a) $\cos(2x+1)$

(b) $\cos(2x-1)$

(c) $\cos(-2x+1)$

(d) $\cos\left(\frac{1}{2}x+1\right)$

(e) $-\cos(2x+1)$

(f) $\sin(2x+1)$

(g) $\sin(2x-1)$.

20. $\arccos(\cos\theta)$ is always defined and $\cos(\arccos\theta)$ is always defined no matter what the value of θ is

   (a) True

   (b) False