

Precalculus Final Exam Spring 2011

Make sure you present your work in a readable and very clean matter. Do
NOT use pen, pencil only

Name

Part 1 (140 points) Each question is worth 20 points

1. (a) **(10 points)** Write the complex number $2 + 2i\sqrt{3}$ in polar form.

(b) **(10 points)** Find the all four 4th roots for $2 + 2i\sqrt{3}$.

2. **(20 points)** Using the n th root theorem, find the 5 distinct solutions to the equation below

$$x^5 = -1$$

You will receive no credit if you solve this equation algebraically. Please follow the instructions.

3. (a) **(10 points)** Write $1 + i$ in polar form

(b) **(10 points)** Using deMoivre's theorem, compute and simplify the following:

$$(1 + i)^{871}$$

4. **(20 points)** Without using a calculator, solve the equation $4 \cos^2 x + 4 \sin x - 5 = 0$ where $0 \leq x \leq 2\pi$

5. **(20 points)** Prove that

$$\tan \theta = \frac{1 - \cos 2\theta}{\sin 2\theta}$$

6. **(20 points)** Without using a calculator simplify $\tan\left(\arccos\frac{3}{5}\right)$

7. **(20 points)** Find the infinite family of solutions to the following trig equation

$$\cos^2(x) = \frac{1}{81}$$

Part 2 (30 points) Each question is worth 3 points

Answer the correct solution in the space provided. Do not show your work just answer the questions.

1. The least positive coterminal of $-\frac{7\pi}{6}$ is:

2. The length of the arc subtended by an angle 60° and radius $r = 3$ is

3. The domain of $\arctan x$ is

4. The radian equivalent of the angle 150° is

5. The quadrants in which $\csc x$ is negative are

6. The phase shift of the harmonic function $3 \sin\left(\frac{1}{3}x - 3\right)$ is

7. Among the following list of functions $\sin x$, $\cos x$, $\tan x$, $\cot x$, $\sec x$, $\csc x$, list the ones which are **even** functions here

8. After simplifying $\arcsin\left(\sin\frac{7\pi}{6}\right)$ we get

9. The graph of the function $\sin(-x)$ is obtained by reflecting the graph of $\sin x$ with respect to the _____ axis.

10. The range of $5\cos(2x - 1)$ written in interval notation is

Part 3 (30 points) Each question is worth 3

1. The graph of the equation $r^2 = 2$ in polar coordinates

- (a) is the graph of a circle of radius 4 (d) is the graph of a circle of radius $1/2$
(b) is the graph of a vector with angle 2 (e) is the graph of a circle of radius $\sqrt{2}$
(c) is the graph of a circle of radius 2 (f) none of the answers is correct

2. The period of the harmonic function $f(t) = 3 \sin(5\pi t - 5)$ is

- (a) $\frac{2\pi}{5}$ (d) $\frac{5\pi}{2}$
(b) $\frac{-2\pi}{5}$ (e) $\frac{-5\pi}{2}$
(c) $\frac{2}{5}$ (f) $\frac{5}{2}$

3. $\sin(x - \frac{7\pi}{2})$ is equal to

- (a) $\sin x$ (d) $-\sin x$
(b) $\cos x$ (e) $\cos x$
(c) $-\cos x$ (f) none

4. $\cos(x - y) \cos(x + y)$ is equal to

- (a) $\cos^2 x \cos^2 y + \sin^2 x \sin^2 y$ (d) $\cos^2 x \cos y - \sin^2 x \sin^2 y$
(b) $\cos^2 x \cos^2 y - \sin^2 x \sin^2 y$ (e) $\cos x \cos^2 y - \sin^2 x \sin^2 y \cos x$
(c) $-\cos^2 x \cos^2 y - \sin^2 x \sin^2 y$ (f) $\cos^2 x \cos^2 y - \sin x \sin^2 y$

5. $\frac{\sin 2x}{2 \sin x \cos x}$ is equal to

- (a) -1 (d) $\cos x$
(b) 1 (e) $-\sin x$
(c) $1/2$ (f) none

6. i^{997} is equal to

- (a) $-i$ (d) i^2
(b) $2i$ (e) i^{-1}
(c) $i/2$ (f) i

7. The exact value of $\cos 75^\circ$ is

- (a) $-\frac{\sqrt{6}-\sqrt{2}}{4}$ (d) $\frac{\sqrt{6}-\sqrt{2}}{4}$
(b) $\frac{\sqrt{4}-\sqrt{2}}{4}$ (e) $\frac{\sqrt{6}+\sqrt{2}}{4}$
(c) $\frac{\sqrt{6}-\sqrt{5}}{4}$ (f) $\frac{\sqrt{6}-\sqrt{2}}{8}$

8. The square roots of i are

- (a) $-\left(\frac{1}{2} - \frac{1}{2}i\right)\sqrt{2}, \left(\frac{1}{2} + \frac{1}{2}i\right)\sqrt{2}$ (d) $-\left(\frac{1}{2} + \frac{1}{2}i\right)\sqrt{2}, \left(\frac{1}{2} + \frac{1}{2}i\right)$
(b) $-\left(\frac{1}{2} + \frac{1}{2}i\right)\sqrt{2}, \left(\frac{1}{2} - \frac{1}{2}i\right)\sqrt{2}$ (e) $-\left(\frac{1}{2} + \frac{1}{2}i\right)\sqrt{3}, \left(\frac{1}{2} + \frac{1}{2}i\right)\sqrt{2}$
(c) $\left(\frac{1}{2} + \frac{1}{2}i\right)\sqrt{2}, \left(\frac{1}{2} + \frac{1}{2}i\right)\sqrt{2}$ (f) $-\left(\frac{1}{2} + \frac{1}{2}i\right)\sqrt{2}, \left(1 + \frac{1}{2}i\right)\sqrt{2}$

9. Solutions to the trig equation $\cos x = -\frac{1}{2}$, are for $k \in \mathbb{Z}$

- (a) $-\frac{2}{3}\pi + 2\pi k, \frac{2}{3}\pi + 2\pi k$ (d) $-\frac{2}{3}\pi + \frac{2}{3}\pi k, \frac{2}{3}\pi + 2\pi k$
(b) $-\frac{2}{3}\pi + \pi k, \frac{2}{3}\pi + \pi k$ (e) $-\frac{2}{3}\pi + \frac{2}{3}\pi k, \frac{2}{3}\pi + \frac{2\pi k}{3}$
(c) $-\frac{2}{3}\pi + \pi k, \frac{2}{3}\pi + 2\pi k$ (f) none of the above

10. In polar coordinates, the graph of $r = e^\theta$

- (a) is a circle (d) a rose with 4 leaves
(b) a rose with 2 leaves (e) a spiral
(c) 2 circles joined together (f) a spiral and a circle