Precalculus Final Exam Spring 2011

Make sure you present your work in a readable and very clean matter. Do NOT use pen, pencil only

Name Key

Part 1 (140 points) Each question is worth 20 points

1. (a) **(10 points)** Write the complex number $2+2i\sqrt{3}$ in polar form.

$$2+2i\sqrt{3} = 4e^{i\pi/3}$$

(b) **(10 points)** Find the all four 4th roots for $2+2i\sqrt{3}$.

$$\varepsilon_0 = \sqrt{2}\, e^{i\pi/12}$$

$$\varepsilon_1 = \sqrt{2}\, e^{i7\pi/12}$$

$$\varepsilon_2 = \sqrt{2}\, e^{i13\pi/12}$$

$$\varepsilon_3 = \sqrt{2}\, e^{i19/12}$$

2. **(20 points)** Using the nth root theorem, find the 5 distinct solutions to the equation below

$$x^5 = -1$$

You will receive no credit if you solve this equation algebraically. Please follow the instructions.

$$x^5 = e^{i\pi} \qquad \xi_k = e^{i\left(\pi/5 + 2k\pi/5\right)}$$

$$\xi_0 = e^{i\pi/5}$$

$$\xi_1 = e^{i3\pi/5}$$

$$\xi_2 = e^{i\pi}$$

$$\xi_3 = e^{i7\pi/5}$$

$$\xi_4 = e^{i9\pi/5}$$

3. (a) **(10 points)** Write $1+i$ in polar form

$$\sqrt{2}e^{i\pi/4} \text{ or } (\sqrt{2}, \pi/4)$$

(b) **(10 points)** Using deMoivre's theorem, compute and simplify the following:

$$(1+i)^{871}$$

$$\left(2^{1/2}e^{i\pi/4}\right)^{871} = 2^{\frac{871}{2}}e^{i\frac{871\pi}{4}}$$

$$= 2^{\frac{871}{2}}e^{i3\pi/4}$$

$$= 2^{871/2}\left(-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)$$

4. **(20 points)** Without using a calculator, solve the equation $4\cos^2 x + 4\sin x - 5 = 0$ where $0 \le x \le 2\pi$

$$4\cos^2 x + 4\sin x - 5 = 0$$

$$4(1-\sin^2 x) + 4\sin x - 5 = 0$$

$$4 - 4\sin^2 x + 4\sin x - 5 = 0$$

$$-4\sin^2 x + 4\sin x - 1 = 0$$

Put $u = \sin x$

$$-4u^2 + 4u - 1 = 0$$

$u = \frac{1}{2}$

$\sin x = \frac{1}{2}$, $x \in [0, 2\pi]$

$$\boxed{x = \frac{\pi}{6},\ x = \frac{5\pi}{6}}$$

5. **(20 points)** Prove that

$$\tan\theta = \frac{1-\cos 2\theta}{\sin 2\theta}$$

$$\frac{1-\cos 2\theta}{\sin 2\theta} = \frac{1-(\cos^2\theta - \sin^2\theta)}{2\sin\theta\cos\theta} = \frac{1-\cos^2\theta+\sin^2\theta}{2\sin\theta\cos\theta}$$

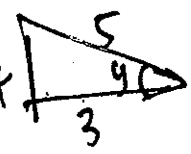
$$= \frac{\sin^2\theta+\sin^2\theta}{2\sin\theta\cos\theta} = \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} = \boxed{\tan\theta}$$

6. **(20 points)** Without using a calculator simplify $\tan\left(\arccos \frac{3}{5}\right)$ $= x$

Put $y = \arccos \frac{3}{5}$, $x = \tan y$

$\cos y = \frac{3}{5} \Rightarrow$



$x = \tan y = \frac{4}{3}$. thus

$$\boxed{\tan\left(\arccos \frac{3}{5}\right) = \frac{4}{3}}$$

7. **(20 points)** Find the infinite family of solutions to the following trig equation

$$\cos^2(x) = \frac{1}{81}$$

$\cos x = \frac{1}{9}$ or $\cos x = -\frac{1}{9}$

$x = \arccos \frac{1}{9} + 2k\pi$, $k \in \mathbb{Z}$

$x = -\arccos\left(\frac{1}{9}\right) + 2k\pi$

$x = \arccos\left(-\frac{1}{9}\right) + 2k\pi$

$x = -\arccos\left(-\frac{1}{9}\right) + 2k\pi$

$$\boxed{\left\{ \arccos \frac{1}{9} + 2k\pi, -\arccos \frac{1}{9} + 2k\pi, \arccos\left(-\frac{1}{9}\right) + 2k\pi, -\arccos\left(-\frac{1}{9}\right) + 2k\pi \right\}}$$

Part 2 (30 points) Each question is worth 3 points

Answer the correct solution in the space provided. Do not show your work just answer the questions.

1. The least positive coterminal of $-\frac{7\pi}{6}$ is:

$\boxed{5\pi/6}$

2. The length of the arc subtended by an angle 60° and radius $r = 3$ is

$\boxed{\pi}$

3. The domain of $\arctan x$ is

$\boxed{(-\infty, \infty)}$ or $\mathbb{R}$

4. The radian equivalent of the angle 150° is

$\boxed{\frac{5\pi}{6} \text{ rad}}$

5. The quadrants in which $\csc x$ is negative are

$\boxed{\text{III}, \text{IV}}$

6. The phase shift of the harmonic function $3\sin\left(\frac{1}{3}x - 3\right)$ is

$\boxed{9}$

7. Among the following list of functions $\sin x, \cos x, \tan x, \cot x, \sec x, \csc x$, list the ones which are **even** functions here

$\boxed{\cos x,\ \sec x}$

8. After simplifying $\arcsin\left(\sin \frac{7\pi}{6}\right)$ we get

$\boxed{-\pi/6}$

9. The graph of the function $\sin(-x)$ is obtained by reflecting the graph of $\sin x$ with respect to the ___y___ axis.

10. The range of $5\cos(2x-1)$ written in interval notation is

$\boxed{[-5, 5]}$

Part 3 (30 points) Each question is worth 3

1. The graph of the equation $r^2 = 2$ in polar coordinates

E

(a) is the graph of a circle of radius 4
(b) is the graph of a vector with angle 2
(c) is the graph of a circle of radius 2
(d) is the graph of a circle of radius 1/2
(e) is the graph of a circle of radius $\sqrt{2}$
(f) none of the answers is correct

2. The period of the harmonic function $f(t) = 3\sin(5\pi t - 5)$ is

(a) $\frac{2\pi}{5}$
(b) $\frac{-2\pi}{5}$
(c) $\frac{2}{5}$
(d) $\frac{5\pi}{2}$
(e) $\frac{-5\pi}{2}$
(f) $\frac{5}{2}$

3. $\sin\left(x - \frac{7\pi}{2}\right)$ is equal to

(a) $\sin x$
(b) $\cos x$
(c) $-\cos x$
(d) $-\sin x$
[redacted]
(f) none

4. $\cos(x - y)\cos(x + y)$ is equal to

(a) $\cos^2 x \cos^2 y + \sin^2 x \sin^2 y$
(b) $\cos^2 x \cos^2 y - \sin^2 x \sin^2 y$
(c) $-\cos^2 x \cos^2 y - \sin^2 x \sin^2 y$
(d) $\cos^2 x \cos y - \sin^2 x \sin^2 y$
(e) $\cos x \cos^2 y - \sin^2 x \sin^2 y \cos x$
(f) $\cos^2 x \cos^2 y - \sin x \sin^2 y$

5. $\frac{\sin 2x}{2\sin x \cos x}$ is equal to

(a) -1
(b) 1
(c) 1/2
(d) $\cos x$
(e) $-\sin x$
(f) none

6. i^{997} is equal to

(a) $-i$
(b) $2i$
(c) $i/2$
(d) i^2
(e) i^{-1}
(f) i

7. The exact value of $\cos 75°$ is

Use the fact that $\cos 75 = \cos\frac{5\pi}{12} = \cos\left(\frac{\frac{5\pi}{6}}{2}\right)$

(a) $-\frac{\sqrt{6}-\sqrt{2}}{4}$ (d) $\frac{\sqrt{6}-\sqrt{2}}{4}$
(b) $\frac{\sqrt{4}-\sqrt{2}}{4}$ (e) $\frac{\sqrt{6}+\sqrt{2}}{4}$
(c) $\frac{\sqrt{6}-\sqrt{5}}{4}$ (f) $\frac{\sqrt{6}-\sqrt{2}}{8}$

8. The square roots of i are

(a) $-\left(\frac{1}{2}-\frac{1}{2}i\right)\sqrt{2}, \left(\frac{1}{2}+\frac{1}{2}i\right)\sqrt{2}$ (d) $-\left(\frac{1}{2}+\frac{1}{2}i\right)\sqrt{2}, \left(\frac{1}{2}+\frac{1}{2}i\right)$
(b) $-\left(\frac{1}{2}+\frac{1}{2}i\right)\sqrt{2}, \left(\frac{1}{2}-\frac{1}{2}i\right)\sqrt{2}$ (e) $-\left(\frac{1}{2}+\frac{1}{2}i\right)\sqrt{3}, \left(\frac{1}{2}+\frac{1}{2}i\right)\sqrt{2}$
(c) $\left(\frac{1}{2}+\frac{1}{2}i\right)\sqrt{2}, \left(\frac{1}{2}+\frac{1}{2}i\right)\sqrt{2}$ (f) $-\left(\frac{1}{2}+\frac{1}{2}i\right)\sqrt{2}, \left(1+\frac{1}{2}i\right)\sqrt{2}$

9. Solutions to the trig equation $\cos x = -\frac{1}{2}$, are for $k \in \mathbb{Z}$

(a) $-\frac{2}{3}\pi + 2\pi k, \frac{2}{3}\pi + 2\pi k$ (d) $-\frac{2}{3}\pi + \frac{2}{3}\pi k, \frac{2}{3}\pi + 2\pi k$
(b) $-\frac{2}{3}\pi + \pi k, \frac{2}{3}\pi + \pi k$ (e) $-\frac{2}{3}\pi + \frac{2}{3}\pi k, \frac{2}{3}\pi + \frac{2\pi k}{3}$
(c) $-\frac{2}{3}\pi + \pi k, \frac{2}{3}\pi + 2\pi k$ (f) none of the above

10. In polar coordinates, the graph of $r = e^{\theta}$

(a) is a circle (d) a rose with 4 leaves
(b) a rose with 2 leaves (e) a spiral
(c) 2 circles joined together (f) a spiral and a circle