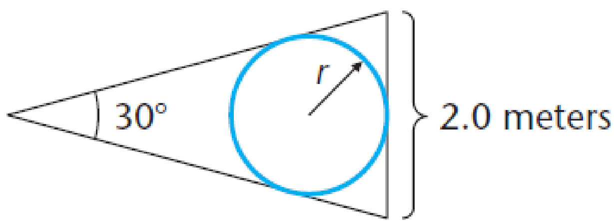
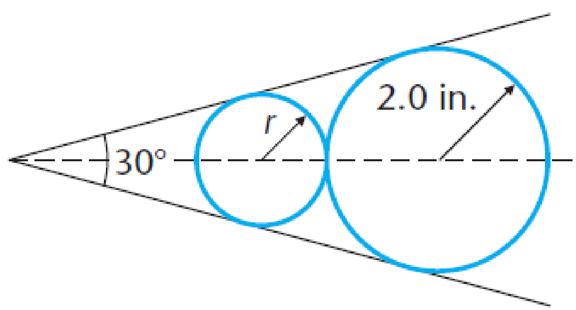
Fall 2011
Precalculus
Project A

This project includes 2 parts. A 10 min class presentation, using powerpoint or overhead which is worth 50 percent of your grade, and a written report which is worth 50 percent of your grade.

1. Find the exact value of r so that the circle is tangent to all three sides of the isosceles triangle. [Hint: The radius of a circle is perpendicular to a tangent line at the point of tangency.]



2. Find the exact value of r so that the smaller circle is tangent to the larger circle and the two sides of the angle



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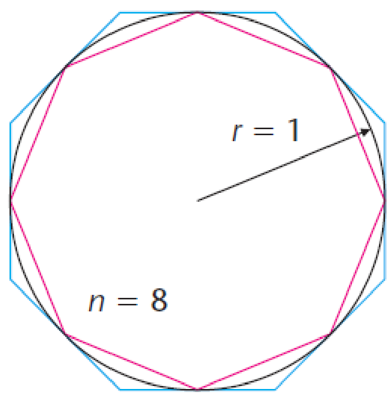
Project B

This project includes 2 parts. A 10 min class presentation, using powerpoint or overhead which is worth 50 percent of your grade, and a written report which is worth 50 percent of your grade.

Show that the area of a regular n-sided polygon circumscribed about a circle of radius 1 is given by $A(n) = n \tan\left(\frac{180^\circ}{n}\right)$

1. Find $A(8), A(100), A(1000)$, and $A(10000)$.
2. What number does A seem to approach as n is getting larger and larger and close to infinity.



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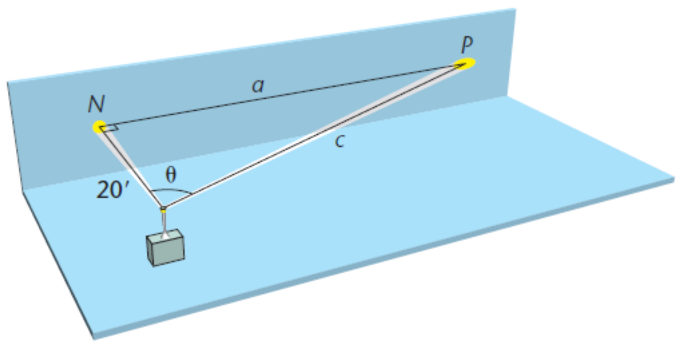
Project C

This project includes 2 parts. A 10 min class presentation, using powerpoint or overhead which is worth 50 percent of your grade, and a written report which is worth 50 percent of your grade.

A beacon light 20 feet from a wall rotates clockwise at the rate of 1/4 revolution per second (rps) (see the figure), therefore, $\theta = \frac{\pi t}{2}$.



1. Start counting time in seconds when the light spot is at N and write an equation for the length c of the light beam in terms of t.
2. Graph the equation found in part 1 for the time interval $[0, 1]$.
3. Describe what happens to the length c of the light beam as t goes from 0 to 1. Explain the behavior of the graph as t is gets closer and closer to infinity.
4. Start counting time in seconds when the light spot is at N and write an equation for the length a of the light beam in terms of t.
5. Graph the equation found in part 4 for the time interval $[0, 1]$.
6. Describe what happens to the length a of the light beam as t goes from 0 to 1. Explain the behavior of the graph as t is gets closer and closer to infinity.

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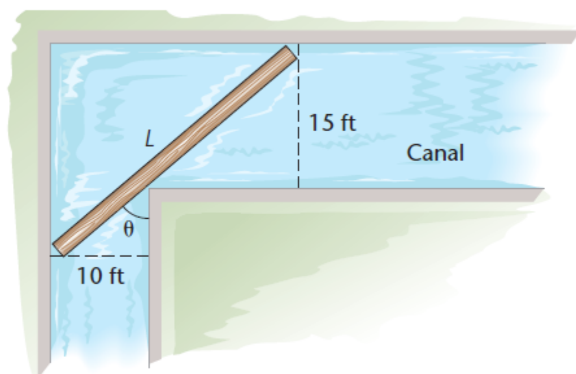
Project D

A 10-foot-wide canal makes a right turn into a 15-foot-wide canal. Long narrow logs are to be floated through the canal around the right angle turn (see the figure). We are interested in finding the longest log that will go around the corner, ignoring the log's diameter.



1. Express the length L of the line that touches the two outer sides of the canal and the inside corner in terms of θ.
2. Find the shortest distance L that is, the length of the longest log that can make it around the corner.
3. Plot the graph of L as a function of θ
4. Explain what happens to the length L as the angle approaches $\frac{\pi}{2}$.

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Project E
Some calculus

Let $F(x) = \sin x$

1. Compute
$$D\left(h\right) = \frac{F\left(\pi + h\right) - F\left(\pi\right)}{h}$$

2. Using your calculator, complete the following table

$$D\left(0.1\right)$$
$$D\left(0.01\right)$$
$$D\left(0.001\right)$$
$$D\left(0.0001\right)$$
$$D\left(0.00001\right)$$

3. Which value is D getting close to as h is approaching zero? Compare your results with $\cos\left(\pi\right)$

4. Compute
$$Q\left(h\right) = \frac{F\left(\pi/2 + h\right) - F\left(\pi/2\right)}{h}$$

5. Using your calculator, complete the following table

$$Q\left(0.1\right)$$
$$Q\left(0.01\right)$$
$$Q\left(0.001\right)$$
$$Q\left(0.0001\right)$$
$$Q\left(0.00001\right)$$

6. What value is Q getting close to as h is approaching zero? Compare your results with $\cos\left(\pi/2\right)$

7. Compute
$$R\left(h\right) = \frac{F\left(\pi/4 + h\right) - F\left(\pi/4\right)}{h}$$

8. Using your calculator, complete the following table

$$\begin{array}{c} R(0.1) \\ R(0.01) \\ R(0.001) \\ R(0.0001) \\ R(0.00001) \end{array}$$

9. What value is R getting close to as h is approaching zero? Compare your results with $\cos(\pi/4)$

10. In general, what kind of conjecture can we make for the value of

$$D(h) = \frac{\sin(x+h) - \sin(x)}{h}$$

as h is getting closer to 0.

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Project F

Consider the function

$$F(x) = \cos(x)^6 - \sin(x)^6.$$

We want to linearize $F(x)$. In other words, we want to write the function F without powers.

1. Establish the identity

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$
$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

2. Using part 1, linearize $\cos^4(x)$ and $\sin^4(x)$.

3. Using part 2, linearize $\cos^6(x)$ and $\sin^6(x)$

4. Finally, show that

$$F(x) = \frac{15}{16}\cos(2x) - \frac{1}{16}\cos(6x)$$

5. Plot both

$$y = \cos(x)^6 - \sin(x)^6$$

and

$$y = \frac{15}{16}\cos(2x) - \frac{1}{16}\cos(6x)$$

together. What do you observe?