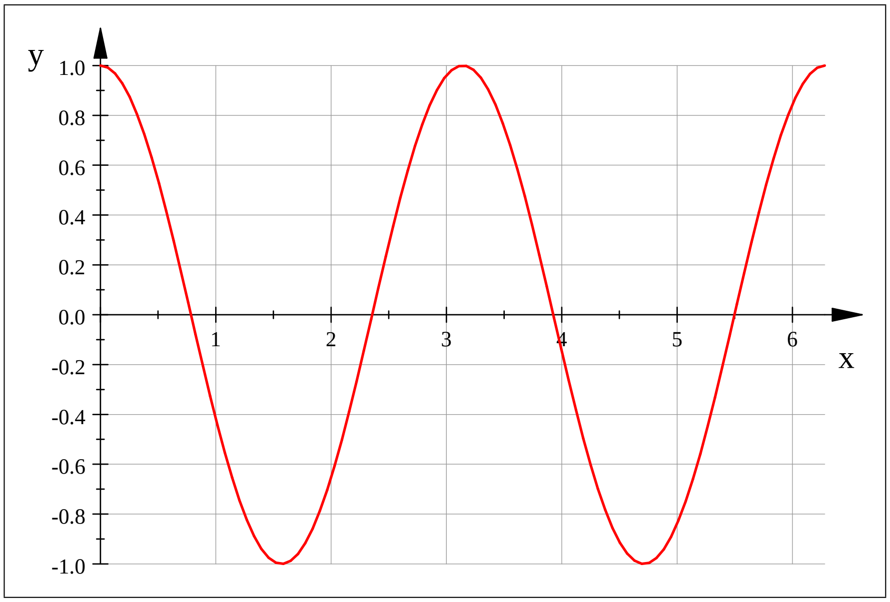
Precalculus Exam 3 Spring 2011 Solutions

1. (10 points) Solve the following trig equation in the interval $[0, 2\pi)$

$$\cos 2x = \frac{\sqrt{2}}{2}.$$

First, let us plot the function $f(x) = \cos 2x$ on the interval $[0, 2\pi)$.



Clearly, we expect to have 4 solutions.

$$\begin{aligned}
\cos 2x &= \frac{\sqrt{2}}{2} \Rightarrow 2x = \frac{\pi}{4} + 2k\pi \text{ or } 2x = \frac{7\pi}{4} + 2k\pi \\
&\Rightarrow x = \frac{\pi}{8} + k\pi \text{ or } x = \frac{7\pi}{8} + k\pi \text{ where } k \in \mathbb{Z} \\
x &= \frac{\pi}{8}, \frac{\pi}{8} + \pi, \frac{7\pi}{8}, \frac{7\pi}{8} + \pi \\
x &= \frac{\pi}{8}, \frac{9\pi}{8}, \frac{7\pi}{8}, \text{ or } \frac{15\pi}{8}
\end{aligned}$$

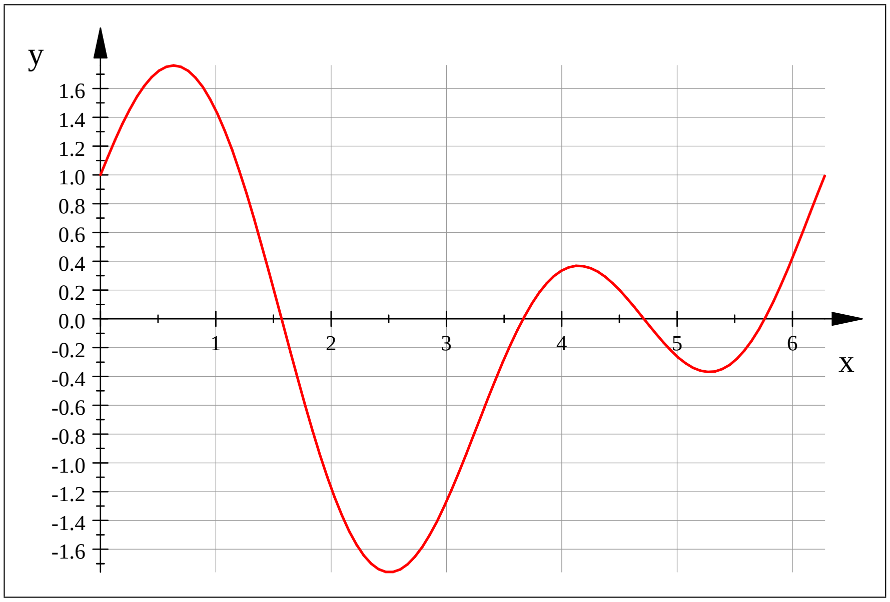
The solution set is

$$\left\{\frac{\pi}{8}, \frac{9\pi}{8}, \frac{7\pi}{8}, \frac{15\pi}{8}\right\}.$$

2. (10 points) Solve in the interval $[0, 2\pi)$ the trig equation

$$\cos x + 2\cos x \sin x = 0.$$

Put $f(x) = \cos x + 2\cos x \sin x$ which we plot first to see where the solutions are located.



$\cos x + 2\cos x \sin x$

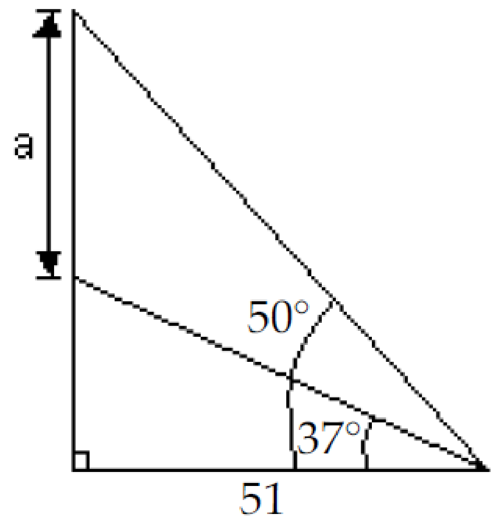
We expect to obtain 4 solutions since we only have 4 x-intercepts.

$$\begin{aligned}\cos x + 2\cos x \sin x &= 0 \Rightarrow \cos x\,(1 + 2\sin x) = 0\\ &\Rightarrow \cos x = 0 \text{ or } 1 + 2\sin x = 0\\ &\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2} \text{ or } x = \frac{7\pi}{6}, \frac{11\pi}{6}.\end{aligned}$$

The solution set is

$$\left\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}.$$

3. (15 points) Find the value of a



$$\begin{aligned} a &= 51\tan(50^\circ) - 51\tan(37^\circ) \\ &= 22.348 \end{aligned}$$

4. (20 points)

a. Find the rectangular coordinates of the following point

$$\left(-3, \frac{3\pi}{4}\right).$$

$$\begin{aligned} x &= r\cos\theta \Rightarrow x = -3\cos\left(\frac{3\pi}{4}\right) = \frac{3}{2}\sqrt{2} \\ y &= r\sin\theta \Rightarrow y = -3\sin\left(\frac{3\pi}{4}\right) = -\frac{3}{2}\sqrt{2} \end{aligned}$$

Thus, we obtain the point of coordinates

$$\left(\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right).$$

b. Find the polar coordinates of the following point

$$\left(4, -4\sqrt{3}\right).$$

$x = 4$ and $y = -4\sqrt{3}$. Thus the point is located in the 3rd quadrant and we compute the modulus of the complex number representing the point given.

$$r = \sqrt{16 + 16\,(3)} = 8$$

and we have that $\theta \in (-\pi, \pi]$

$$\begin{aligned} \tan\theta &= \frac{y}{x} = \frac{-4\sqrt{3}}{4} = -\sqrt{3} \Rightarrow \theta = \arctan\left(-\sqrt{3}\right) \\ &\Rightarrow \theta = -\pi/3 \end{aligned}$$

Thus we obtain

$$(8, -\pi/3).$$

5. (10 points) Convert the rectangular equation to a polar equation that expresses r in terms of θ

$$x^2 + y^2 = 16$$

The solution is $r^2 = 16$

6. (10 points) Convert the polar equation to a rectangular equation. **Hint**: use the double angle formula.

$$r^2 \sin(2\theta) = 4.$$

$$\begin{aligned} r^2 \sin 2\theta &= r^2 2 \sin\theta \cos\theta \\ &= 2\,(r \sin\theta)\,(r \cos\theta) \\ &= 2yx \end{aligned}$$

Thus we have

$$\begin{aligned} 2yx &= 4 \\ y &= \frac{2}{x}. \end{aligned}$$

7. (15 points) First fill in the tables below as done in class, and plot in the space provided below the tables the following expression given in polar coordinates:

$$r = 3\cos 3\theta$$

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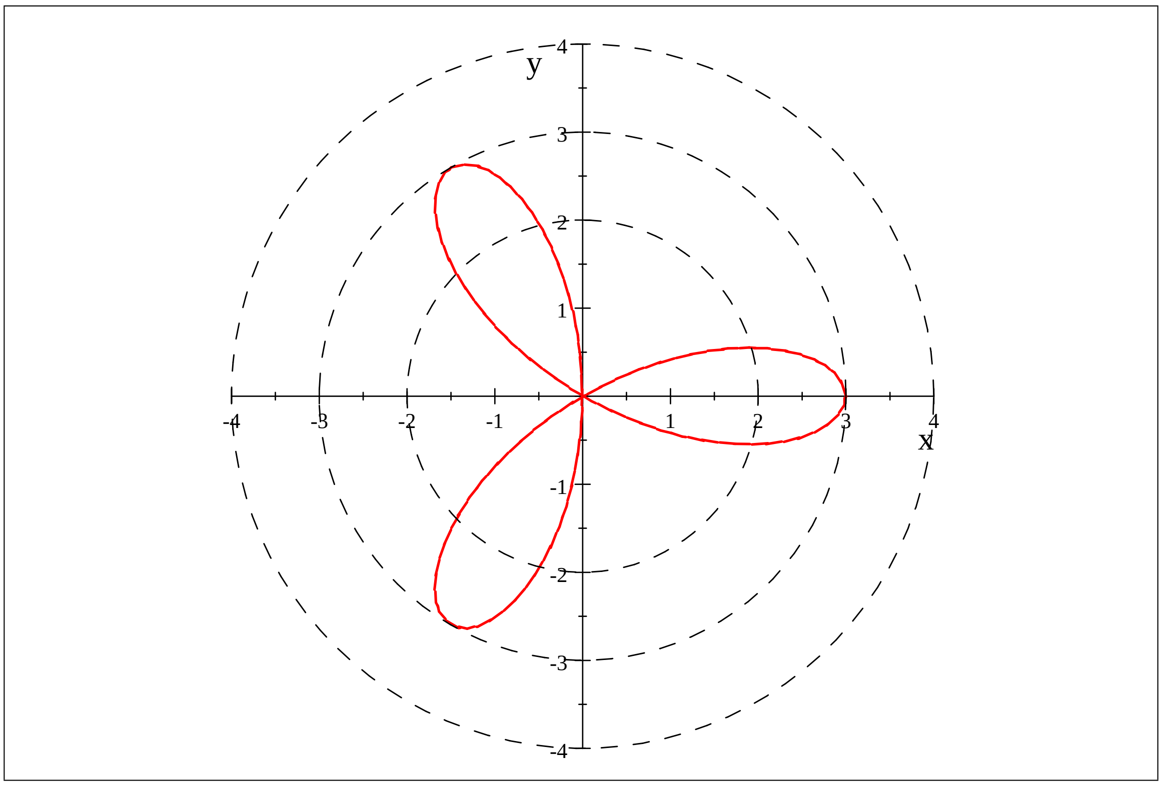

You must plot your graph here !

8. (10 points) Write the complex number in rectangular form.(recall the rectangular for is of $x + iy$)

$$5\left[\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right]$$

$$= 5\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$

$$= \frac{5}{2}i\sqrt{3} - \frac{5}{2}$$