A.1 Algebra Essentials

• Work with sets

A set is a well-defined collection of objects.

If a set is empty, we call it a null set or empty set, we use the symbol

$\{\ \}$, ϕ

$D = \{1, 2, 3, 5, 7, 9\}$

D is written is roster notation

Ex 1 (Set Builder notation)

$E = \{x \mid x \text{ is an even digit}\}$

Set builder notation

$E = \{0, 2, 4, 6, 8\}$

Roster Notation.

Ex Write $E = \{1, 3, 5, 7, 9\}$ in set builder notation.

$\boxed{E = \{x \mid x \text{ is an odd digit}\}}$

Def A set A is a subset of a set B if every element of A is an element of B. We write $A \subseteq B$.

Ex $A = \{1, 2, 3, 4\}$

$B = \{1, 3, 4, 5, 7\}$

Is A a subset of B?

No, because 2 is not an element of B.

Ex $A = \{1, \frac{1}{2}\}$, $B = \{\frac{1}{2}, 1, -1\}$

Is $A \subseteq B$? True.

Ex Let A, B be 2 sets

The intersection of A and B denoted $A \cap B$ is the set consisting of elements that belong to both A and B.

Ex $A = \{1, 2, -1\}$, $B = \{1, -1, \frac{1}{2}\}$

$$A \cap B = \{1, -1\}$$

Def The union of A with B denoted $A \cup B$, is the set consisting of elements that belong to either A or B both.

Ex $A = \{1, 2, 3\}$, $B = \{-1, 1, 5\}$

$$A \cup B = \{1, 2, 3, -1, 5\}$$

Def A universal set is the set under discussion. It is denoted U.

Ex If we are talking about numbers, then the universal set is the set of all numbers.

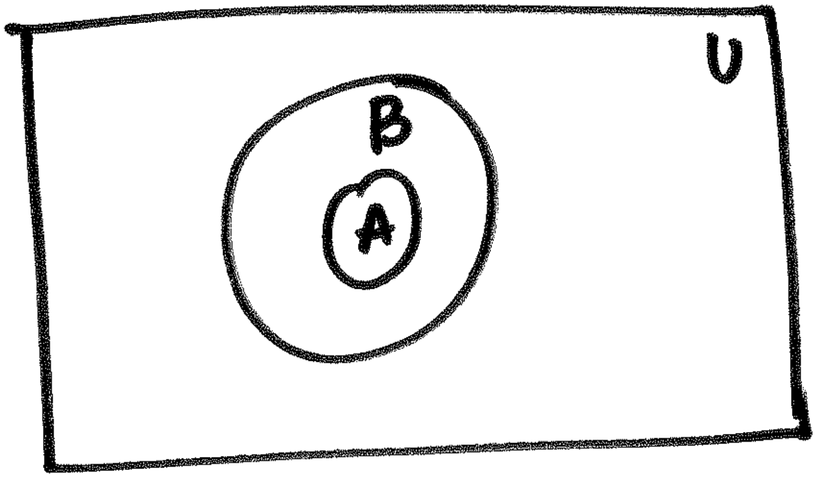
Def Let A be a set. The complement of A is denoted $\bar{A}$ and $\bar{A}$ is the set of all elements in the universal set not in A.

Ex $U = \{1, 2, 3, 4, 5, 6\}$

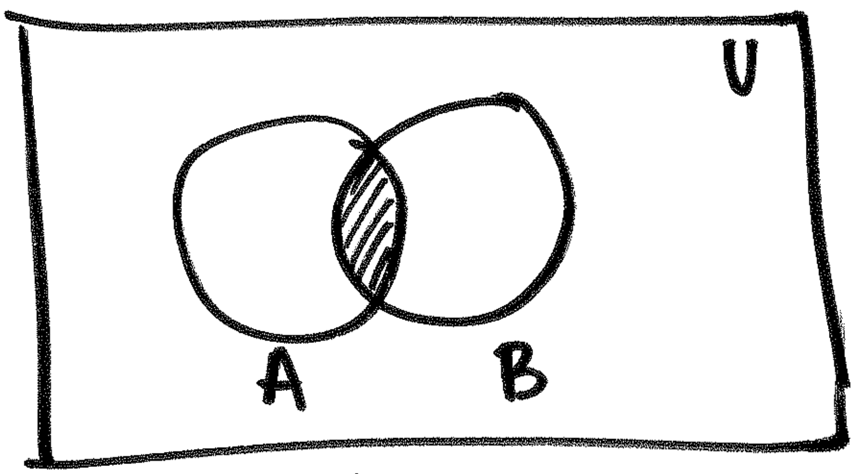
$A = \{2, 4, 6\}$

$\bar{A} = \{1, 3, 5\}$

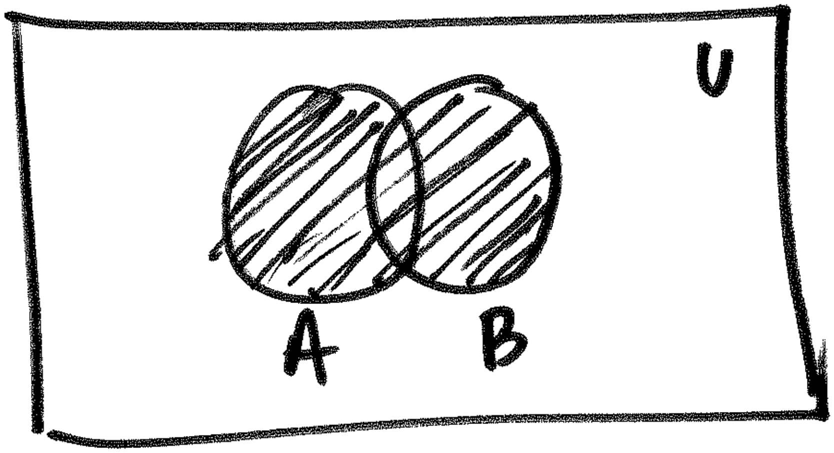
<u>Diagram</u> Venn Diagram



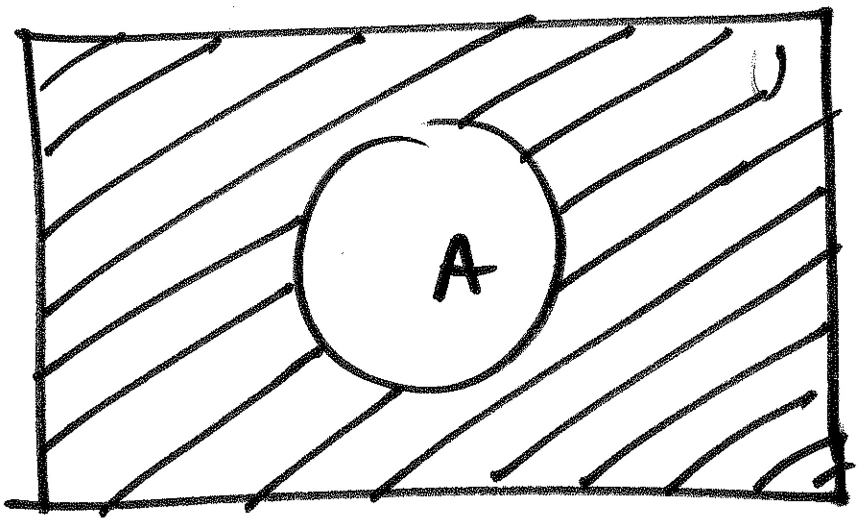
$A \subseteq B$



$A \cap B$



$A \cup B$



$\overline{A}$

Real numbers

Distributive Property

$$a \cdot (b+c) = ab + ac$$

Zero-product Rule

$$a \cdot b = 0 \text{ if } a = 0 \text{ or } b = 0.$$

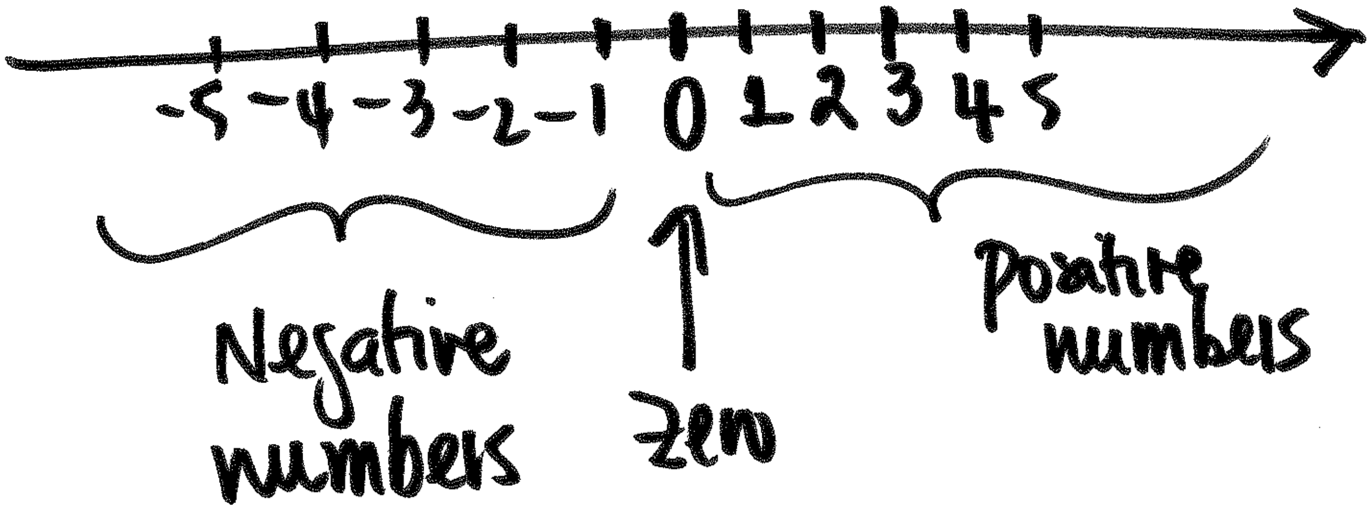
Ex Find x if $5x = 0$

$$\cancel{5=0} \text{ or } x = 0$$

Ex $$2(5+3) = 2 \times 5 + 2 \times 3$$
$$= 10 + 6 = 16$$

Ex $$\frac{1}{2}(1-2) = \frac{1}{2} - \frac{1}{2}(2)$$
$$= \frac{1}{2} - 1 = \boxed{-\frac{1}{2}}$$

The real number line

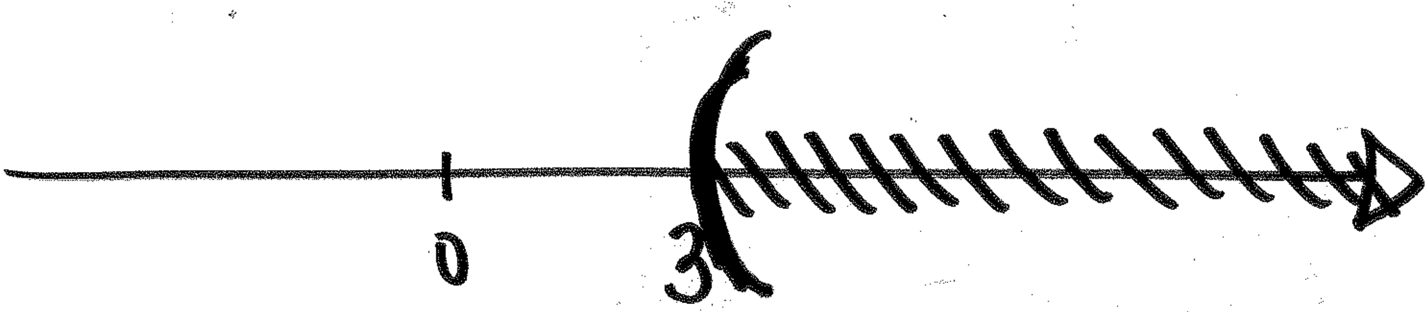


Absolute Value

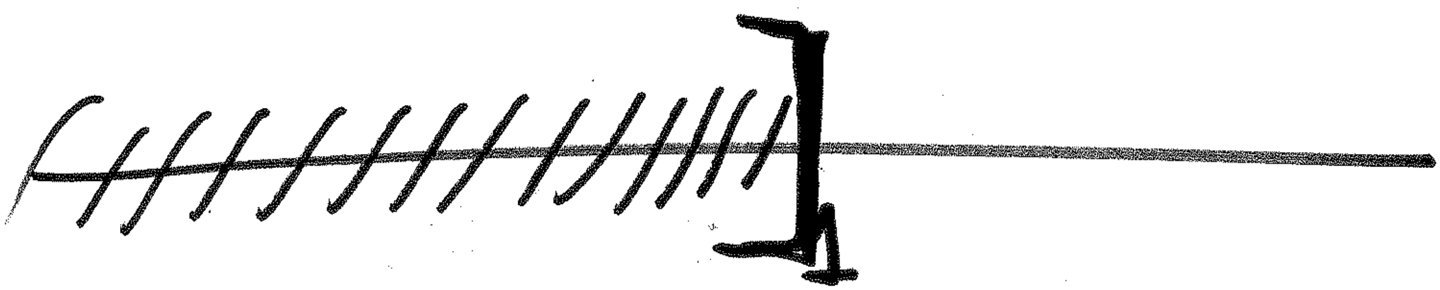
$$|a| = a \quad \text{if } a \geq 0$$

$$|a| = -a \quad \text{if } a \leq 0$$

Ex Graph on the real line the set of all x such that $x > 3$

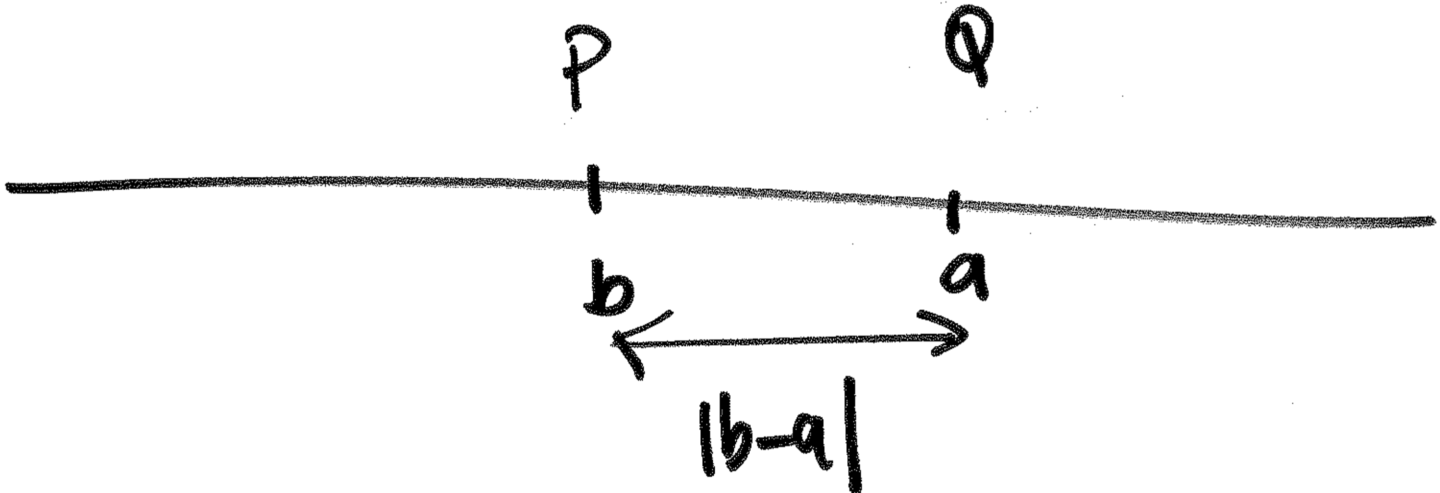


Ex Graph the set of all x such that $x \leq 1$



<u>Def</u> Let P, Q be pts on the real line with coordinates, a, b

$$d(P, Q) = |b - a|$$



<u>Ex</u> Find the distance between 3 and -4

$$|3-(-4)| = |3+4| = |7| = 7$$

Ex Evaluate an Algebraic Expression

Evaluate the expression below if

$x = 1, \quad y = 2$

$$\left| \frac{2x}{x+y} \right| - 1$$

$$= \left| \frac{2 \cdot 1}{1+2} \right| - 1 = \left| \frac{2}{3} \right| - 1$$

$$= \frac{2}{3} - 1 = \frac{2}{3} - \frac{3}{3} = \boxed{-\frac{1}{3}}$$

Ex Find the domain of

$$\frac{5}{x-2}$$

$D = \{ x \mid x \neq 2 \}$

Ex Find the domain of

$$\frac{5}{x(x-1)}$$

$D = \{ x \mid x \neq 0 \text{ and } x \neq 1 \}$

Ex **Laws of exponents**

$$2^3 = 2 \cdot 2 \cdot 2 = 8$$

$$\left(\frac{1}{2}\right)^4 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$$

Def $$a^n = \underbrace{a \cdot a \cdot a \cdots \cdot a}_{n \text{ factors}}$$

1. $a^0 = 1$, $a \neq 0$

2. $a^{-n} = \dfrac{1}{a^n}$ if $a \neq 0$

Ex $$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

$$\left(\frac{1}{4}\right)^{-2} = \frac{1}{\left(\frac{1}{4}\right)^2} = \frac{1}{\frac{1}{16}} = 16$$

Laws of Exponents

$$a^m a^n = a^{m+n}$$

$$(a^m)^n = a^{m \cdot n}$$

$$(ab)^n = a^n b^n$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Ex

$$\frac{x^5 y^{-2}}{x^3 y} = \frac{x^2}{y^3}$$

Ex $\left(\frac{x^{-3}}{3y^{-1}}\right)^{-3} = \frac{x^9}{3^{-3} y^3}$

$$= \frac{27x^9}{y^3}$$

__Def__ If a is a nonnegative real number, the nonnegative number b such that $b^2 = a$ is called the principal square root of a and is denoted by $\sqrt{a}$.

__Ex__ $\sqrt{4} = 2$ because $2^2 = 4$

$\sqrt{\frac{1}{9}} = \frac{1}{3}$ because $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$

__Remark__ $\sqrt{a^2} = |a|$