Operations on functions

DEFINITION 1 Operations on Functions

The **sum**, **difference**, **product**, and **quotient** of the functions f and g are the functions defined by

$$(f+g)(x) = f(x) + g(x) \qquad \text{Sum function}$$

$$(f-g)(x) = f(x) - g(x) \qquad \text{Difference function}$$

$$(fg)(x) = f(x)g(x) \qquad \text{Product function}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \qquad g(x) \neq 0 \qquad \text{Quotient function}$$

The **domain** of each function consists of all elements in the domains of *both* f and g, with the exception that the values of x where $g(x) = 0$ must be excluded from the domain of the quotient function.

Example

1. Let $f(x) = \sqrt{4-x}$ and $g(x) = \sqrt{3+x}$. Find $f+g$ and its domain.
2. Let

$$f(x) = \frac{1}{x+2}$$

and

$$g(x) = \frac{x-1}{x}.$$

Find $\frac{f}{g}$ and find its domain.

Composition of functions

We define $\boxed{(f \circ g)(x) = f(g(x))}$

Example. Given $f(x) = x - 1$ and $g(x) = (x+1)^2$

1. Compute $(f \circ g)(1)$
2. Compute $(g \circ f)(1)$
3. Compute $(g \circ f)(x)$
4. Compute $(f \circ g)(x)$
5. Compare the functions $f \circ g$ with $g \circ f$
6. Express h as a composition of two simpler functions for $h(x) = \sqrt{1+x^2}$