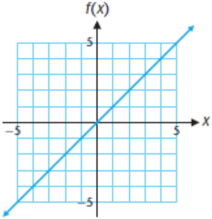
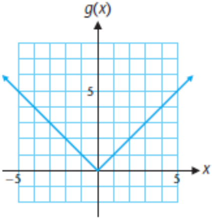
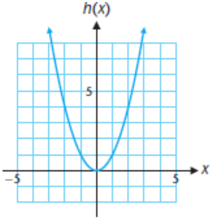
A Library of Elementary Graphs



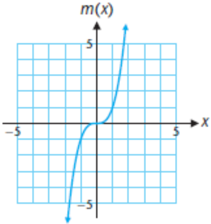
(a) Identity function
$f(x) = x$
Domain: R
Range: R



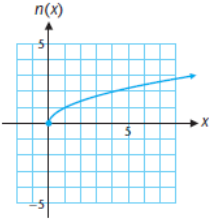
(b) Absolute value function
$g(x) = |x|$
Domain: R
Range: $[0, \infty)$



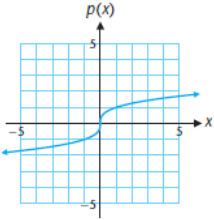
(c) Square function
$h(x) = x^2$
Domain: R
Range: $[0, \infty)$



(d) Cube function
$m(x) = x^3$
Domain: R
Range: R



(e) Square root function
$n(x) = \sqrt{x}$
Domain: $[0, \infty)$
Range: $[0, \infty)$



(f) Cube root function
$p(x) = \sqrt[3]{x}$
Domain: R
Range: R

SUMMARY OF GRAPH TRANSFORMATIONS

Vertical Translation [Fig. 12(a)]:

$$y = f(x) + k \qquad \begin{cases} k > 0 & \text{Shift graph of } y = f(x) \text{ up } k \text{ units} \\ k < 0 & \text{Shift graph of } y = f(x) \text{ down } |k| \text{ units} \end{cases}$$

Horizontal Translation [Fig. 12(b)]:

$$y = f(x + h) \qquad \begin{cases} h > 0 & \text{Shift graph of } y = f(x) \text{ left } h \text{ units} \\ h < 0 & \text{Shift graph of } y = f(x) \text{ right } |h| \text{ units} \end{cases}$$

Vertical Stretch and Shrink [Fig. 12(c)]:

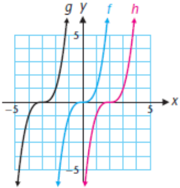
$$y = Af(x) \qquad \begin{cases} A > 1 & \text{Vertically stretch the graph of } y = f(x) \text{ by multiplying each } y \text{ value by } A \\ 0 < A < 1 & \text{Vertically shrink the graph of } y = f(x) \text{ by multiplying each } y \text{ value by } A \end{cases}$$

Horizontal Stretch and Shrink [Fig. 12(d)]:

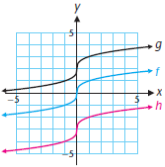
$$y = f(Ax) \qquad \begin{cases} A > 1 & \text{Horizontally shrink the graph of } y = f(x) \text{ by multiplying each } x \text{ value by } \frac{1}{A} \\ 0 < A < 1 & \text{Horizontally stretch the graph of } y = f(x) \text{ by multiplying each } x \text{ value by } \frac{1}{A} \end{cases}$$

Reflection [Fig. 12(e)]:

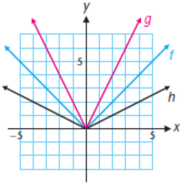
$y = -f(x)$ Reflect the graph of $y = f(x)$ in the x axis
$y = f(-x)$ Reflect the graph of $y = f(x)$ in the y axis
$y = -f(-x)$ Reflect the graph of $y = f(x)$ in the origin



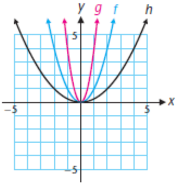
(b)
Horizontal translation
$g(x) = f(x + 3)$
$h(x) = f(x - 2)$



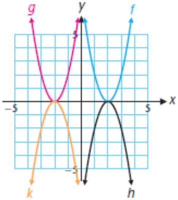
(a)
Vertical translation
$g(x) = f(x) + 2$
$h(x) = f(x) - 3$



(c)
Vertical stretch and shrink
$g(x) = 2f(x)$
$h(x) = 0.5f(x)$



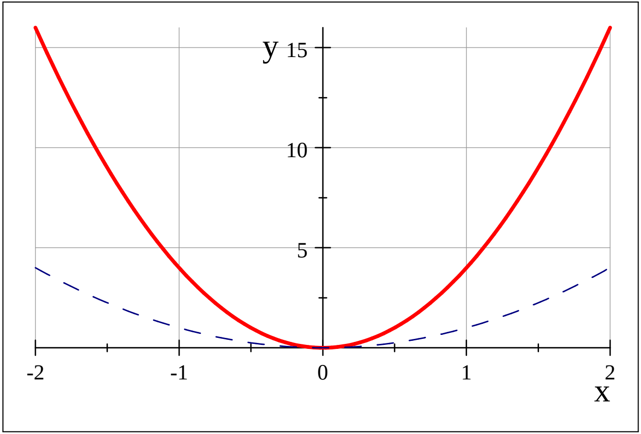
(d)
Horizontal stretch and shrink
$g(x) = f(2x)$
$h(x) = f(0.5x)$



(e)
Reflection
$g(x) = f(-x)$
$h(x) = -f(x)$
$k(x) = -f(-x)$

Examples

1. Using graph transformations, sketch the graph of $g(x) = -(x-2)^2$
2. Using the graph transformation , sketch the graph of $f(x) = (2x)^2$ and $g(x) = x^2$ together

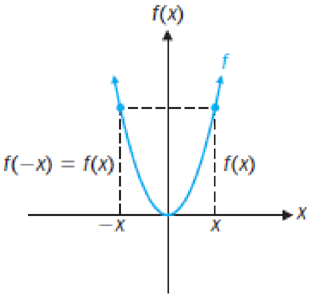


Even and odd functions

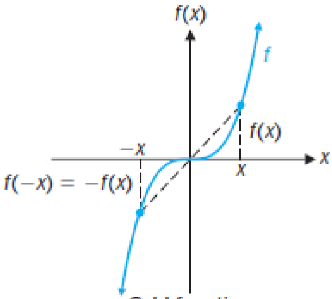
If $f(x) = f(-x)$ for all x in the domain of f, then f is an **even function.**

If $f(-x) = -f(x)$ for all x in the domain of f, then f is an **odd function.**

The graph of an even function is said to be **symmetric with respect to the y axis** and the graph of an odd function is said to be **symmetric with respect to the origin**



Even function
(symmetric with
respect to y axis)



Odd function
(symmetric with
respect to origin)

Even and odd functions

Examples Determine whether the functions f, g, and h are even, odd, or neither.

1. $f(x) = x^4 + x^2$
2. $g(x) = x^3 + 1$
3. $h(x) = x^3 + x$
4. $k(x) = \frac{1}{x}$

Example The graph of the function g is formed by applying the indicated sequence of transformations to the given function f. Find an equation for the function g. Check your work by graphing f and g in a standard viewing window.

1. The graph of $f(x) = |x|$ is reflected in the x axis, vertically shrunk by a factor of 0.5, shifted three units to the right, and shifted four units up.
2. The graph of $f(x) = x^3$ is shifted five units to the right and four units up.